Adults learning mathematics — inside and outside the classroom

June 29th — July 2nd 2014

Proceedings of the 21st International Conference of Adults Learning Mathematics — A Research Forum (ALM)

Edited by Anestine Hector-Mason and Sonja Beeli-Zimmermann

ALM

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Hosted by the University of Bern, the Swiss Federal Institute for Vocational Education and Training SFIVET and the Swiss Federation for Adult Learning SVEB
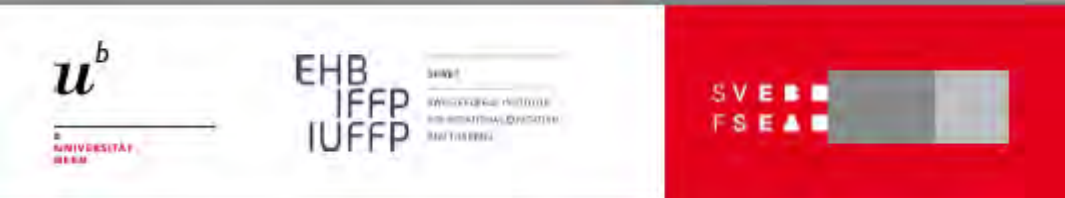
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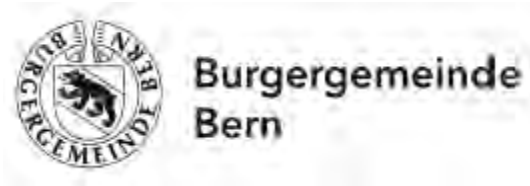
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About ALM

Adults Learning Mathematics – A Research Forum (ALM) was formally established in July, 1994 as an international research forum with the following aim:

- To promote the learning of mathematics by adults through an international forum that brings together those engaged and interested in research and development in the field of mathematics learning and teaching.

Charitable Status

ALM is a Registered Charity (1079462) and a Company Limited by Guarantee (Company Number: 3901346). The company address is: 26 Tennyson Road, London NW6 7SA.

Aims of ALM

ALM's aims are promote the advancement of education by supporting the establishment and development of an international research forum for adult mathematics and numeracy by:

- Encouraging research into adults learning mathematics at all levels and disseminating the results of this research for the public benefit.

- Promoting and sharing knowledge, awareness and understanding of adults learning mathematics at all levels, to encourage the development of the teaching of mathematics to adults at all levels for the public benefit.

ALM's vision is to be a catalyst for the development and dissemination of theory, research and best practices in the learning of mathematics by adults, and to provide and international identity for the profession through an international conference that helps to promote and share knowledge of adults' mathematics teaching and learning for the public benefit.

ALM Activities

ALM members work in a variety of educational settings, as practitioners and researchers, to improve the teaching and learning of mathematics at all levels. The ALM annual conference provides an international network which reflects on practice and research, fosters links between teachers, and encourages good practice in curriculum design and delivery using teaching and learning strategies from all over the world. ALM does not foster one particular theoretical framework, but encourages discussion on research methods and findings from multiple frameworks.

ALM holds an **international conference** each year at which members and delegates share their work, meet each other, and network. ALM produces and disseminates Conference Proceedings and a multi-series online Adults Learning Mathematics – International Journal (ALMJ).

On the ALM website http://www.alm-online.net, you will also find pages of interest for teachers, experienced researchers, new researchers and graduate students, and policy makers.

- **Teachers:** The work of members includes many ideas for the development and advancement of practice, which is documented in the Proceedings of ALM conferences and in other ALM publications.

- **Experienced Researchers:** The organization brings together international academics, who promote the sharing of ideas, publications, and dissemination of knowledge via the conference and academic refereed journal.

- **New Researchers and Ph.D. Students:** ALM annual conferences and other events allow a friendly and interactive environment of exchange between practitioners and researchers to examine ideas, develop work, and advance the field of mathematics teaching and learning.

- **Policymakers:** The work of the individuals in the organization helps to shape policies in various countries around the world.

ALM Members

ALM Members live and work all over the world. See the ALM members' page at www.alm-online.net for more information on regional activities and representatives, and for information on contacting your local membership secretary.

How to become a member: Anyone who is interested in joining ALM should contact the membership secretary. Contact details are on the ALM website: www.alm-online.net.

Membership fees for 2014:

	Sterling	**Euro**	**US Dollar**
Individual	20	24	32
Institution	40	48	80
Student/unwaged	4	6	8
Low waged	Contribute between full and unwaged		

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ALM is managed by a Board of Trustees elected by the members at the Annual General Meeting (AGM), which is held at the annual international conference.

Adults Learning Mathematics – Inside and Outside the Classroom

ALM Proceedings Supervising Editor: Anestine Hector-Mason, Ph.D

ALM 21 Proceedings Editors: Anestine Hector-Mason, Ph.D. and Sonja Beeli-Zimmerman

ALM 21 Local Organizer: Sonja Beeli-Zimmerman

These proceedings include both refereed and non-refereed papers. Peer reviewed papers are indicated with an asterisk (*) next to the title in the table of contents.

Photo front cover: University of Bern

Photos back cover: Sonja Beeli-Zimmermann

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Acknowledgement

The ALM is grateful to the many people who have contributed to the ALM 21 conference and to the production of these proceedings:

- The ALM members who contributed to both the conference and the proceedings for ALM 21.

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- ALM Officers and Trustees for ensuring the continuity of the organization between the various conferences.

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Preface: About ALM 21

The 21st international conference of Adults Learning Mathematics – A Research Forum (ALM 21) was held in Bern Switzerland. The conference was organized by the University of Bern in collaboration with the Swiss Federal Institute for Vocational Education and Training SFIVET and the Swiss National Umbrella Organization for Adult Education SVEB. It was attended by researchers, practitioners and policymakers from 12 nations (Canada, France, Ireland, the Netherlands, New Zealand, Norway, South Africa, Spain, Sweden, Switzerland, the United Kingdom and the United States of America). In addition to its traditionally diverse and inspiring program of presentations and workshops, this conference featured a number of 'firsts,' which are indicative for ALM's longer term objectives and goals, namely:

- A half-day pre-conference academic writers' workshop which aimed at supporting particularly new researchers and practitioners in their writing process and thereby contributed to ALM's goal of developing an inclusive research community.

- Simultaneous translation of the third ALM conference day into German in order to include many Swiss practitioners – a multilingual experiment indicative of ALM's strive for a diverse and encompassing membership.

The theme of the conference was 'Adults Learning Mathematics Inside and Outside the Classroom', which called attention to the practice of mathematics in the classroom, and in other spheres of life. It was chosen, not least of all, in order to reflect a specificity of the Swiss educational system: its duality. Accordingly, the conference program, and especially the key note presentations, reflected (a) this separation in physical terms and (b) how learning activities on both sides of a classroom wall can be coordinated:

- The first day was dedicated to the inside of the classroom and raised some overarching issues, namely the evidence base for teaching (keynote by Walter Herzog) and multilingual contexts (keynote by Mamokgethi Phakeng).

- The second day focused on learning outside of the classroom and with experiences from in-company trainings (keynote by Cäcilia Märki) and the presentation of a mathematics learning app (keynote by Susan Easton) which had a very practical perspective.

- On the third day the relationship between two physically separate locations and their associated knowledge stood at the center. Coordinating separate experiences (keynote by Hansruedi Kaiser) as well as different types of skills and knowledge (keynote by Corinne Hahn) were examined, and provided deeper insight into the Swiss and the French perspectives, respectively.

As you, the reader, will see from the submissions, presentations, and abstracts in these Proceedings, the practice and conduct of mathematics do not only occur in square rooms in school buildings. Various forms of technology transport mathematical content to the four walls of one's home, for example, in the form of a flipped classroom, or accompany us in the form of a mobile phone app. On the other hand, artefacts or practical tasks, such as creating a bag from leather skin, bring professional situations into the classroom. The borders between the two are not nearly as clear as the conference theme makes seem – mathematics is all around us.

On a more formal note it is worth mentioning that these are the first proceedings which ALM publishes with an official open access license. Explicitly including such as license formally

reflects ALM's spirit of sharing information and best practices, as it allows for unrestricted access to the materials and therefore increases ALM's visibility.

Note: All conference presentation submitted for publication and meeting editors' requirements for style and presentation were reviewed and are published in the ALM 21 Conference Proceedings. Presentations for which no paper was submitted are represented by an abstract or power point slides. As there are a considerable number of power point slides, the format of the submissions has also been taken into account in the organization of these proceedings: Both the keynotes as well as the other contributions are presented in two sections, namely a first section with articles and a second section with power point slides. In addition there is one section with posters and abstracts. A coloured pdf version of the Proceedings is available on ALM's website (http://www.alm-online.net/alm-conference-proceedings/).

SECTION 1

Keynotes

Section 1.a.

Keynote Speaker Articles

The French 'Alternance' Model: The Question of the Relationship Between Professional Skills and Academic Knowledge

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Abstract

In this paper I will first present the French 'Alternance' Model which is a way to conceptualise vocational education, in particular apprenticeship. 'Alternance' Model, as defined by Geay (1998), is a four dimensional model. I will explore the didactic dimension, centred on knowledge. I will discuss the notion of 'real problem,' and I will present pedagogical devices designed for apprentices. I will give examples of my work in mathematics education with young adults working in the trade sector: shop assistants, merchandisers, sales managers.

Keywords: alternance, device, logic, workplace, system, objects, pragmatic concepts

French 'Alternance' Model

Geay (1998) defined 'alternance' as an interface system in between two systems, school and workplace (see figure 1). These two systems have two different logics, different ways of learning, and different ways to consider knowledge. His 'Alternance' Model has four dimensions: institutional (administrative organisation), personal (about learners' identity), pedagogic (organization of the formal learning process), and didactic (link between disciplinary knowledge and experienced-based knowledge). The model works only if the four interlinked dimensions are activated.
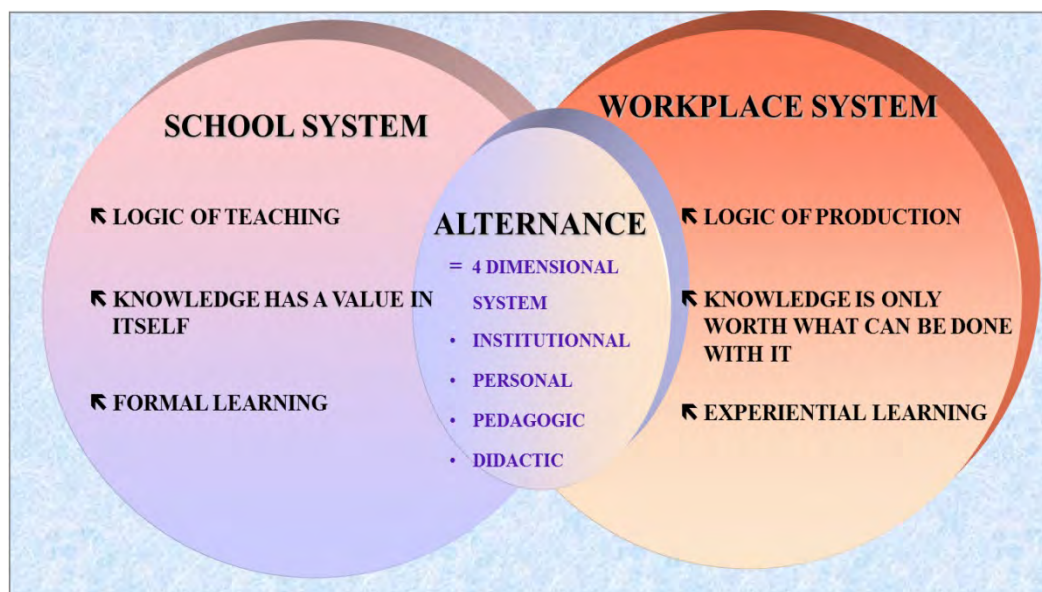

Figure 1. 'Alternance' model (Geay, 1998)

This systemic approach is deeply rooted in French culture, because of the importance given to knowledge and also because the approach is centred on the learner. The learner is at the heart of the system and not just one element of it (as in Engeström's model of expansive learning). However, it is also in opposition to French tradition as there is no hierarchy between different forms of knowledge. Many researchers, referring to different frameworks, share the conception of the learning process as a dialectical process between conceptualization in action and theoretical concepts and not as a simple vertical process directed toward mastery of theoretical knowledge.

I draw on a socio-constructivist framework based on Gerard Vergnaud's theory of conceptual fields. For Vergnaud, the locus of learning is located in the individual, although he recognizes the importance of the socio-cultural environment. According to him, a concept is linked to a set of situations that give meaning to that concept. Operational invariants, such as theorem-in-action and concepts-in-action, play an important role in the process of adapting to new situations. Vergnaud claims that we need to analyse professional activity and identify its conceptual components. According to him, knowledge has two forms: an operational/practical one and a predicative/theoretical one. He claims that alternance gives the opportunity for each form to support the development of the other.

To explain this seminal notion of operational invariant, I will give an example from a research I conducted with shop-assistants (Hahn, 2000). I observed how they used percentages at school and in the workplace. I saw that, most of the time, to calculate a price without a 18.6% tax, they 'subtract' 18.6% instead of divide by 1.186. This is a very common and resistant mistake, an epistemological obstacle (Bachelard, 1970). Referring to Vergnaud's theory, apprentices used the theorem-in-action 'x% is inverse of +x%' associated with the concept-in-action that % is a unit.

'Real' Problems, Boundary Objects, and Pragmatic Concepts

We know that school and workplaces have different epistemologies (Noss et al, 2000) and it is not enough to recommend the use of 'real' problems or 'authentic' situations (Adda, 1976). We have to consider vertical and horizontal processes (Bakker, 2014).

It is often recommended to use 'real' problem in order to help learners to link what they learn at school with what they experienced in and out of school situations. But this is not enough, and it can even have negative effects as I will show in another example from my work with shop assistants. I observed that to calculate a 20% discount, they [the assistants] systematically multiply by 0.8 and then subtract the net price from the gross price, instead of just multiplying by 0.2. Of course, this method is not incorrect, but it is longer. I observed that this knowledge is rooted in local professional practices and reinforced by school practices. In fact, in the shop the assistants never calculate the amount of a discount; they always calculate the net price. At school, teachers ask them to calculate the net price to fit to workplace practices. As a consequence, students no longer know how to calculate the amount of a discount directly (e.g., multiplying by 0.2 to calculate 20%. An excessive use of 'real' workplace problems at school may hinder the development of more general knowledge. It leads to the creation of didactical obstacles, due to school practices.

A way to overcome this difficulty could be to use boundary objects (Star & Griesemer, 1989; Akkerman & Bakker, 2011). Boundary objects are artefacts used in the workplace that also make sense at school. The use of boundary objects facilitates communication between different communities, in this case - school and workplace. To help students to overcome the obstacle of additive percentage, I used different boundary objects. These boundary objects were instructions sent to shop owners by their professional unions when the tax rate decreased from 22% to 18.6%. For example, I used a poster sent to jewellery shops presenting the tax change as a discount: '2.3% on marked prices'. The jewellery shop apprentices were tasked with finding out what calculation was hidden behind this marketing trick and how to adapt it to other situations.

Another notion I integrated in my work is the notion of pragmatic concept. This notion was defined by Pierre Pastré (1998) who builds the field of 'professional didactics' drawing on Vergnaud's work, Russian psychology and French ergonomy. This notion is inspired by Vygotsky's everyday concept,

but it has a smaller range of validity as it is restricted to the professional field. Pragmatic concepts are forms of conceptualization that help to organize efficient action. They are linked to a class of situations and the relation between pragmatic concepts and scientific concepts is dialectic. Pastré claims that didactical situations based on pragmatic concepts enhance professional development. Here is an example of a pragmatic concept and how I used it:

> *Some years ago I was asked by an international soft drinks company to design a mathematics course for their merchandisers. I interviewed merchandisers and I studied their argumentation with department supervisors. I realised that merchandisers' strategies depended on how they conceptualise the situation. To sell a new product, a seasonal merchandiser first makes a diagnosis by evaluating the 'room'; then he adapts negotiation techniques. Department managers are usually reluctant to reference a new product because it is necessary to reorganize the shelves. If the merchandiser identifies a space big enough for the product by moving the others on the shelves, then he has good chances to convince the manager. Expertise in merchandising reflects awareness of this notion of 'room'. Evaluating the 'room' requires knowing about area and volume calculations and spatial representations, which was not expected. Note that in French we have two different words for 'shelves': 'étagères' in a general sense and 'linéaire' in retailing. 'Linéaire' is already a geometrical conceptualization of what shelves represents in retailing.*

> *As this pragmatic conceptualization of room is central in the success of negotiation I designed a pedagogical situation based on it. Students were asked to prepare a negotiation strategy between a merchandiser and a department supervisor using the map of the department and the shelves, information on products, on competitors' sales etc.*

Design

A design experiment is a specific pedagogical device which intends to address complex problems in education through the design of a learning environment interwoven with the testing of theory (Cobb et al, 2003, Bakker & Van Eerde, 2013). I built a design experiment in statistics to get some insight about how students link different types of conceptualizations and, at the same time, design a device that could be used to support the learning process.

The following is a short description of a design experiment I conducted with 36 postgraduate business students (Hahn, 2014). Most of them had previous work experience as salespersons and had applied for a master's degree to become sales managers. The problem on which the pedagogical device was built was an authentic situation based on a real firm and describing an event that really happened. Students had to choose between three sales areas in order to get the job of a sales manager. They were provided with an excel file of data collected on a sample of customers (firms) in each area. The pedagogical device involved four steps, including building a database.

To build the database I conducted a literature which included an a priori analysis (Artigue, 1988) in order to identify statistical obstacles. Through the literature review, it appeared that students seldom use numerical summaries (Konold & Pollatsek, 2002), and if they calculated summaries they make no use of common sense to answer statistical problems (Bakker, 2004). Students have a natural strategy to study extremes and to divide in subgroups (Hammerman et Rubin, 2004; Noss et al, 2000) and have difficulty to move from a local to a global point of view and to construct the concept of distribution (Makar & Confrey, 2005). For clarifications, it is important to consider 2 types of variation: within and between groups (Garfield & Ben-Zvi, 2005), as is shown in Figure 2.

I anticipated that through this device or model (see figure 2) students would have to move from a local to a global point of view and that, to choose the best area, they had to connect knowledge from both worlds and integrate statistics in decision making. The research results showed that the use of statistical summaries was limited, decreased as students move forward in the experiment, and was dependent on the context (the meaning of the variable) and not only on the distribution of numbers. In fact students were not solving the same problem as the procedure they used was primarily related to

the personal experience they associated with the problem. Students' conceptions seem to be related to three forms of rationality: a technical rationality (application of techniques which are not put into perspective); a pragmatic rationality (use of intuitive strategies to meet a limited short-term objective); and a scientific/epistemic rationality (integration of theories to enlighten the problem).

The Device
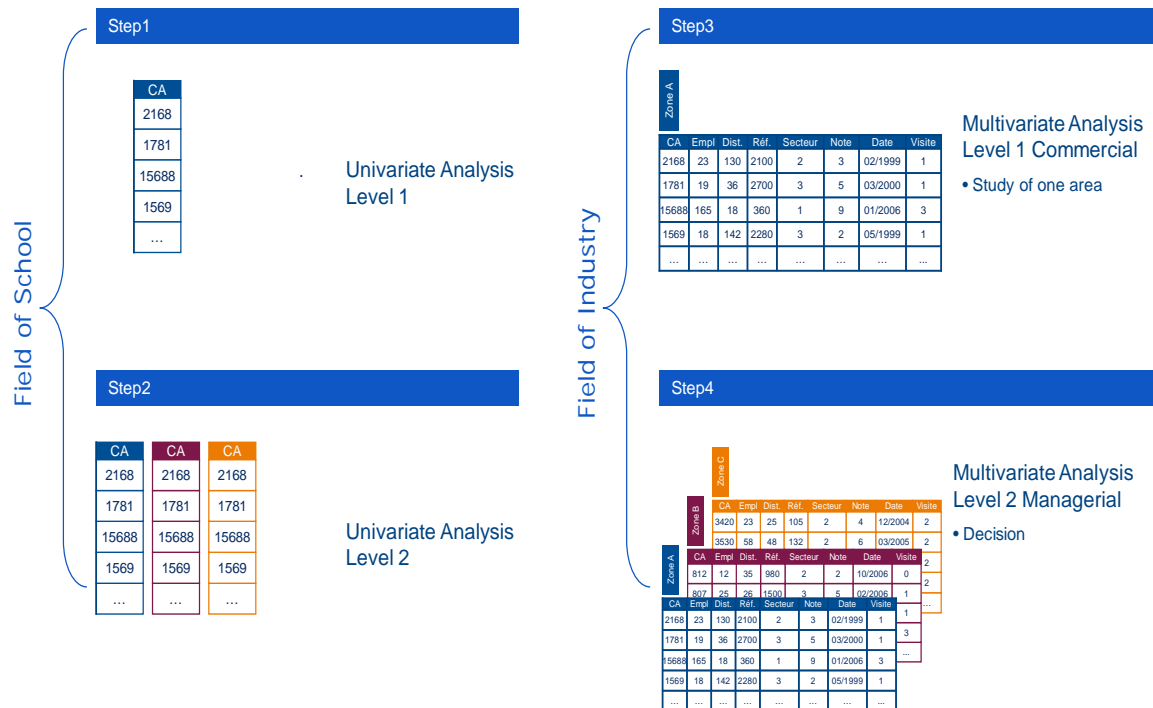
Figure 2. The device

Although scientific rationality is the rationality that schools typically referred to, pragmatic rationality usually prevailed. It seems to me that each form of rationality is linked to a dominant identity: technical is linked to a student's identity, pragmatic to a salesman's identity, and scientific more to a manager's identity. Abreu (2000) claims that the resistance in using some specific knowledge can be explained by the fact that some identities are less valued than others. Indeed, the identity of the salesperson seemed to be more valued than the identity of a student. The challenge is to make students understand that both identities must converge if they want to gain access to the new identity of a manager. The few students who built a more managerial approach to the problem seem to question and link knowledge of different origins, trade and statistics, by emancipating themselves from the roles they had previously constructed.

Conclusion

In this paper, I described some pedagogical devices in mathematics and statistics designed to helps learners to better link theoretical knowledge with professional knowledge. Apprenticeship gives the opportunity to design meaningful pedagogical devices, in order to make students confront different forms of rationality and help them to 'web' conceptualisations at different levels. This leads me back to the four interlinked dimensions of the 'Alternance' Model. The institutional organization of apprenticeship allowed me to design a pedagogical experiment that helps students to bridge

knowledge from different origins and supports the construction of their professional identity. Then the four interlinked dimensions were activated and that is why the device had positive effects. If one believes, as I do, that mathematics education has an important role to play in Adult Education, then it is important to understand how mathematics is developed and recontextualized by learners (FitzSimons, 2014), how it contributes to the construction of the learner's identity, and how it allows the learner to understand the challenges and the changes of activities in different contexts. According to me, that is what the role of school should be.

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Can there be an Evidence Base for Mathematics Teaching?

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Abstract

In this paper the idea of an evidence base for teaching is critically examined on the basis of contributions that educational psychology made to the understanding of teaching and learning. The author argues that teaching in general and teaching mathematics in particular cannot be conceived of as having an evidence base that prescribes how to teach in a technological sense, but only a scientific base, which demands competencies like intuition, improvisation and judgement to be used constructively by teachers.

Keywords: evidence base, mathematics, teaching, technology

Introduction

I have been invited to open this conference – even though I am neither a mathematician nor an adult educator. I am a simple educational psychologist who – however – is confronted time and again with the question of how his discipline can be put to practical use. This is the origin of the title of my presentation, which aims at comprehensively framing the learning – or more precisely the teaching – of mathematics.

The organisation behind this conference is focusing on *learning* mathematics – more specifically *adults* learning mathematics. To me, this name expresses a certain scepticism towards teaching – particularly institutionalised teaching.

Traditionally, educators have a hard time imagining learning without teaching. John Dewey can give us a vivid illustration thereof. In his classic *How we think*, which is still worth reading today, he writes:

> Teaching may be compared to selling commodities. No one can sell unless someone buys. We should ridicule a merchant who said that he had sold a great many goods although no one had bought any. […] There is the same exact equation between teaching and learning that there is between selling and buying (Dewey, 1938; 1989, p. 35f.).

If only – one is tempted to say, particularly in the context of mathematics education. But precisely the teaching of mathematics shows that Dewey cannot be right. While there is a logic or a semantic relationship between selling and buying, the relationship between teaching and learning is *not* a logical but an empirical one and therefore a contingent one. It is probably no coincidence that this forum, which leaves the relationship between teaching and learning open in its name, exists in the field of mathematics.

However, teaching *does* play a role, as it is also implied by the theme of this conference: Adults Learning Mathematics – Inside and Outside the Classroom. At least *inside* the classroom not only learning, but also teaching is taking place. And insofar as today's focus is on 'Inside the classroom', the theme of my presentation, which will focus on teaching, might not be as unsuitable as it might seem at first sight.

Nevertheless, this should not stop me from taking seriously the scepticism towards teaching, which is expressed in ALM's naming – although it is probably less a scepticism towards teaching, but rather a certain reservation towards a premature *linking* of teaching and learning.

This seems all the more important, as even in the field of empirical classroom research we have found ourselves confronted with models which pass off teaching as causal for learning. Already the classical process-product paradigm used in classroom research is based on a simplistic, linear causal model. The same can be said about the model of educational productivity developed by Herbert Walberg or about the various input-output models, which are riding high in the standards movement or international large-scale assessments.

Moreover, in the German speaking context, a flood of terms has established itself in recent years, virtually promoting the same reductionism: Words like teaching-learning situation, teaching-learning setting, teaching-learning arrangement, even teaching-learning methods, teaching-learning activity and teaching-learning process imply that we are dealing with a homogenous, self contained occurrence, meaning that teaching and learning are connected like a system of communicating tubes.

Learning not only seems to be *directly* attached to teaching, it also seems to emerge causally from teaching. Where teaching is done the right way, the message is, successful learning inevitably will take place. By directly linking the teachers' teaching with the learners' learning – at the conceptual or empirical level – all practical problems of classroom teaching, including the mathematics classroom, seem to dissolve miraculously.

This brings me to the topic of my presentation. Because in its own way evidence-based education is also promising that learning can be directly linked to teaching if only its recommendations are followed. My presentation is structured as follows:

1. In a first step I will illustrate what is meant by evidence-based education.
2. In a second step I will present arguments against the idea of an evidence base by looking into my discipline's history.
3. In a third step I will try to systematise these objections by drawing a distinction between complexity and complicatedness.
4. Finally, I will conclude my presentation with a short outlook.

What is Meant by Evidence-Based Education?

What is meant by evidence-based education respectively evidence-based teaching? Do not worry, I will not go into details. We all have heard that we should follow the example set by medicine and align our actions as educators to scientific evidence. Just like many other terms in the field of school and education, which quickly lose their conceptual clarity once they hit the ground of everyday life, the concept of evidence-based education is already showing signs of wear. This is countered by the fact that evidence is ascribed to the term evidence itself. We cannot help but taking the call for basing educational actions on evidence as evident. We are therefore confronted with the triviality that also the field of education should consider results obtained by scientific research.

However, the question framing my presentation is not about this triviality. This is why I would like to briefly outline what I have in mind when I talk about basing teaching on evidence. My point of reference is Robert Slavin. In a paper which he presented at the 2002 annual meeting of the American Educational Research Association he laments the backwardness of educational research. While other

sciences had reached the 21st century a long time ago, educational research is just about to leave the 19th:

> At the dawn of the 21st century, education is finally being dragged, kicking and screaming, into the 20th century. The scientific revolution that utterly transformed medicine, agriculture, transportation, technology, and other fields early in the 20th century, almost completely bypassed the field of education (Slavin, 2002, p. 16).

Even though this quote talks about a *scientific* revolution, Slavin's position is not rooted in science or the philosophy of science, but in educational practice. The examples provided by medicine, agriculture, transportation and technology are examples of applying scientific results. His lamentation is therefore not about science or scientific insights themselves, but about a science who seemingly has nothing to say about the improvement of practice.

You can call this a Baconian understanding of science. Francis Bacon demanded science to unequivocally contribute to the improvement of the human condition. Scientific progress is technological progress which is at the same time human progress. It is the objective of science 'to endeavour to renew and enlarge the power and empire of mankind in general over the universe' (Bacon, 1620, p. 129). According to evidence-based education this should also apply to the educational universe.

So what is up for discussion is the *practical relevance of science*, an issue that goes hand in hand with a specific understanding of practice, which is hidden behind the concept of effectiveness. The progress which Slavin reports in relation to agriculture, medicine and technology should also find its way into the educational system, meaning that education and teaching should be as effective as the work of farmers, physicians and engineers.

So when we ask, what evidence-based teaching means, then the answer is that first of all it is about practice – evidence-based practice – and secondly it is about the effectiveness of this practice – evidence of effectiveness. The purpose of scientific research is to provide this evidence.

In order to assess whether an educational programme is effective, we need causal knowledge. The method of choice when it comes to generating causal knowledge is – according to the creed of most psychologists – the experiment, which is why the evidence demanded by Slavin can only be gained if educational research is adopting rigorous experimental research methods:

> […] the experiment is the design of choice for studies that seek to make causal conclusions, and particularly for evaluations of educational innovations. Educators and policymakers legitimately ask, 'If we implement Program *X* instead of Program *Y*, or instead of our current program, what will be the likely outcomes for children?' For questions posed in this way, there are few alternatives to well-designed experiments (Slavin, 2002, p. 18).

Slavin continues by noting that 'Once we have dozens or hundreds of randomized or carefully matched experiments going on each year on all aspects of educational practice, we will begin to make steady, irreversible progress' (p. 19).

So when I ask whether teaching mathematics can be based on evidence, the answer will be determined by this understanding.

Arguments against Evidence-Based Education

I will not try to present a systematic criticism of evidence-based education. Rather I would like to take a walk through the history of educational psychology to identify existing arguments for countering this technological understanding of educational practice. My walk will start at the end of the 19th century and end in the beginning of the 21st.

Talks to Teachers

In 1899 William James published his famous *Talks to Teachers on Psychology* which he previously presented to various audiences in numerous places. The first chapter carries the title 'Psychology and the Teaching Art'. By addressing himself directly to the teachers, he expresses his suspicion that they might expect him to provide specific information about mental processes, which would enable them 'to labor more easily and effectively in the several schoolrooms over which you preside' (James, 1899, p. 5). However, James believes that he is not able to fulfil these expectations. He admittedly is far away from disclaiming

> For psychology all title to such hopes. Psychology ought certainly to give the teacher radical help. And yet I confess that, acquainted as I am with the height of some of your expectations, I feel a little anxious lest, at the end of these simple talks of mine, not a few of you may experience some disappointment at the net results. In other words, I am not sure that you may not be indulging fancies that are just a shade exaggerated (p.5)

One could be under the impression that James had his discipline's youthfulness and its still inadequate findings it had to offer in mind. But this is only partly true, because his main argument is another. It would be a great, indeed a very great mistake, he says,

> If you think that psychology, being the science of the mind's laws, is something from which you can deduce definite programmes and schemes and methods of instruction for immediate schoolroom use. *Psychology is a science, and teaching is an art*; and sciences never generate arts directly out of themselves. An intermediary inventive mind must make the application, by using its originality (p.7 – my emphasis, W.H.).

James calls on logic and ethics as analogies. Never until now has logic taught a person right judgement, just as little as ethics has not yet lead anyone to virtuous actions. While science is able to set boundaries, it cannot say what we ought to do *within* these boundaries.

> A science only lays down lines within which the rules of the art must fall, laws which the follower of the art must not transgress; but what particular thing he shall positively do within those lines is left exclusively to his own genius. One genius will do his work well and succeed in one way, while another succeeds as well quite differently; yet neither will transgress the lines (p. 8).

According to James, a teacher's educational action needs to comply with psychology – nothing more and nothing less. In order to be a good teacher, one therefore needs a certain talent,

> a happy tact and ingenuity to tell us what definite things to say and do when the pupil is before us. That ingenuity in meeting and pursuing the pupil, that tact for the concrete situation, though they are the alpha and omega of the teacher's art, are things to which psychology cannot help us in the least (p. 9).

James then makes use of a metaphor which might surprise us: He compares the art of teaching to the art of war. While here and there the principles are 'simple and definite' their application is difficult, particularly because there exists an unpredictable counterpart whose intentions and plans are not easily accessible.

Technology of Teaching

James presented his *Talks to Teachers* before behaviourism rose to power in American psychology. If we were to identify a predecessor for the evidence movement, it was behaviourism. Skinner, whose self-conception as a scientist is known to stand in the tradition of Francis Bacon, advocated a position diametrically contrary to that of William James, namely a psychology which is able to *directly* guide teachers' actions. Skinner also talks about 'The Art of Teaching', but what he means by that is something quite different from James. *His* art is technology in disguise – accordingly the book's title in which Skinner's essay was published reads: *The Technology of Teaching* (1968).

Skinner beliefs that a technology of teaching can be immediately deduced from the science of learning: 'Education', so Skinner says, 'is perhaps the most important branch of scientific technology' (Skinner, 1968, p. 19). So important, in fact, that Skinner used his insights about controlling the learning process, to create teaching machines. Just like Slavin, Skinner envisions us at the threshold of a revolution in our educational system.

> There is no reason why the schoolroom should be any less mechanized than, for example, the kitchen. A country which annually produces millions of refrigerators, dishwashers, automatic washing machines, automatic clothes driers, and automatic garbage disposers can certainly afford the equipment necessary to educate its citizens to high standards of competence in the most effective way (Skinner, 1968, p.27)

Scientific Basis of the Art of Teaching

Thankfully we have moved on from behaviourism, but not from the technological concepts upon which it is based, which brings me to my next station on my walk through the history of educational psychology: Nathaniel Gage. In 1978 Gage's title *The Scientific Basis of the Art of Teaching* was published in which he presented an overview of where empirical education research stood at the time. Just like William James' book also this publication was based on presentations which Gage had given, namely at the Teacher's College at Columbia University.

In the first chapter he explains the title of his book. He differentiates between a 'science of teaching' and a 'scientific basis for the art of teaching'. 'The former idea, a science of teaching, claims much more and is in the end, I think, erroneous. It implies that good teaching will some day be attainable by closely following rigorous laws that yield high predictability and control' (Gage, 1978, p. 17).

Gage considers this to be impossible for reasons we already have heard of from William James. Practical actions require virtuosity or in the words of Gage 'intuition, creativity, improvisation, and expressiveness' (p. 15), meaning the readiness and ability to deviate from given schemata, rules and formulae. As mentioned before, Gage's reasoning is similar to that of James: The teaching situation is too complex, particularly with respect to its social dynamics, which is why teachers depend on judgement, intuitive insight, sensitivity and presence of mind.

In his argumentation Gage is taking a further step by pointing out that not only teachers, but also physicians and engineers depend on their judgement, as practical actions generally contains both 'artistic elements, [and] a scientific base' p. 18). Professional action is not limited to applying covering laws. It also contains the element of 'knowing when to follow the implications of the laws, generalizations, and trends, and, especially, when *not* to, and how to combine two or more laws or trends in solving a problem' (p. 18).

Donald Schön (1983) further elaborated this understanding of professional practice some years later in his work *The Reflective Practitioner* – however without referring to Gage.

Gage did not change his position in his later years. Even in his posthumously published *A Conception of Teaching* he quotes a phrase from the previously mentioned book and points to the necessity of experience as basis for teaching:

> The teacher will learn from experience when she should stay close to the implications of the covering laws and when to depart from them. And she will learn from experience whether the structure of the present theory helps her think constructively about her teaching. (Gage, 2009, p. 149)

Covering laws are useful, but they cannot be applied in a simplified manner. Without experience it is impossible to adequately use scientific evidence.

Evidence Does not Speak for Itself

I will take another leap in time for my third example and by doing so will step into the proverbial lion's den. Even if you have never heard of evidence-based education (what I do not think), you will most certainly have heard of John Hattie.

It is a rare occurrence for English research literature on school and teaching to be translated into German. Apart from the previously mentioned book by Gage which was published in 1978 and translated into German one year later, another publication which met this fate was *Fifteen Thousand Hours* by Michal Rutter and his colleagues. Published in 1979 it was also translated into German one year later. I cannot recall any other translations ever since. But Hatties' *Visible Learning* is another example of an English book about school and teaching research that is considered to be so important by many that it was translated into German last year.

The book about which the *Times Educational Supplement* said it would reveal the Holy Grail of teaching (Mansell, 2008), is clearly situated in the field of evidence-based education. Already on the first page of the English edition it states that *Visible Learning* represents 'the largest ever collection of evidence-based research into what actually works in schools to improve learning' (Hattie, 2009, p. I). However, Hattie not only wants to know what works, he wants to know what works *best*. The preface states: 'The major message [of this book, W.H.] is that we need a barometer of what works best [for students, W.H.]' (p. IX – accentuation removed). What surprises most is that Hattie presents this message in the form of a story.

In the book's reception this aspect has not yet been considered adequately (cf. Herzog, 2014). Effect sizes and their presentation in rankings of determinants of student learning stand in the foreground – even though already the book's title indicates that Hattie aims for more, because 'visible learning' is a metaphor. And for Hattie this metaphor contains a message more important than listing effect sizes.

Evidence alone is not enough, he tells us:

> Certainly it could be claimed that more than 800 meta-analyses based on many millions of students is the epitome of 'evidence based' decision making. But the current obsession with evidence-based too often ignores the lens that researchers use to make decisions about what to include as evidence, what to exclude, and how they marshal the evidence to tell their story. It is the *story* that is meant to be the compelling contribution – it is my lens on this evidence (Hattie, 2009, p. 237 – his emphasis).

The lens through which he examines the multitude of meta-analyses is a story: the story of visible teaching and learning. 'The major argument is that when teaching and learning is visible, there is a greater likelihood of students reaching higher levels of achievement' (p. 38).

This means that the story of visible teaching and learning, which Hattie tells us, is not derived from his hundreds of meta-analyses, but serves – as lens – to make sense of them in the first place. Without the story all those effect sizes would stand like lone trees in which we could not see a forest called teaching. This perspective is all the more remarkable as Hattie acknowledges that his story is by no means the only one possible. 'The 'story' told in this book about visible teaching and visible learning is one set of plausible hypotheses to fit a model to these data and the data to the model – there are certainly many more' (p. 248).This statement provides a strong relativisation of the concept of evidence-based education – ironically by an exponent of the very movement.

If we were to measure the evidence movement by its own yardstick, namely the guidance of educational practice, then it is precisely Hattie who shows us that in a strong sense this is impossible. He says the following about the relationship between evidence and practice: 'Evidence does not provide us with rules for action but only with hypotheses for intelligent problem solving, and for making inquiries about our ends of education' (Hattie, 2009, p. 247). His message, therefore, is not that teachers should orient themselves along the listed effect sizes, but rather that they use the reported

results as hypotheses for intelligent problem solving by conducting their own research in their own classes. With this message, however, Hattie is far closer to James and Gage than to Skinner and Slavin.

The Essence of the Argument

Our short walk through the history of educational psychology has shown us that the idea of basing teacher actions on evidence can by no means be considered to be evident. The arguments found in James', Gage's and Hattie's writings carry too much weight against an evidence basis for education for us to be bewitched by Slavin's siren songs. Still open, however, is the question, what precisely constitutes the core of this criticism. This is what I would like to illustrate now.

I have mentioned the analogy between the art of teaching and the art of war used by William James in order to illustrate his position. Interestingly, Lee Shulman uses a similar analogy almost 90 years later. Shulman has, as you probably know, conducted a series of studies examining the work contexts of teachers and physicians and in doing so also compared the two groups. Time and again you can find clear statements in his writings regarding the high complexity of teaching.

In a festschrift for Nathaniel Gage with the telling title *Talks to Teachers* one can find the following: 'If there is any kind of medicine that resembles teaching, it may be emergency medicine on the battlefield' (Shulman, 1987, p. 384). With this Shulman makes clear that a teacher in a classroom is exposed to much higher degrees of complexity than a physician examining a patient – unless he finds himself in the emergency room: 'When 30 patients want your attention at the same time, only then do you approach the complexity of the average classroom on an average day' (Shulman, 2004, p. 504). Shulman is convinced 'that classroom teaching […] is perhaps the most complex, most challenging, and most demanding, subtle, nuanced, and frightening activity that our species has ever invented' (p. 504). He might be slightly exaggerating, but his keyword is essential: *complexity*. Complex situations demand more than technological actions than ever imagined by the evidence movement.

Complexity needs to be differentiated from complicatedness. In an essay entitled *Science and Complexity*, the mathematician Warren Weaver discusses three kinds of problems: problems of simplicity, problems of disorganized complexity und problems of organized complexity. The first group of problems is the subject of classical physics, the second can be accessed with statistical analysis while the third presents an enormous challenge to the sciences. It includes problems 'which involve dealing simultaneously with a sizeable number of factors which are interrelated into an organic whole' (Weaver, 1948, p. 539). It is unlikely that Weaver thought of teaching when writing this, yet a school class is a perfect example for his term of organised complexity.

Unfortunately, Weaver does not tell us how to deal with problems of organised complexity. Rather his essay concludes with reflections about the boundaries of science. Science has presented impressive results for dealing with problems of simplicity, whereas the hard problems, namely the problems of organized complexity, still lie ahead.

It is obvious that classroom research is dealing with this last kind of problems in Weaver's sense. David Berliner confirms this, although he does not talk about easy and hard *problems*, but about easy-to-do and hard-to-do *science*. 'Easy-to-do science is what those in physics, chemistry, geology, and some other fields do. Hard-to-do science is what the social scientists do and, in particular, it is what we educational researchers do' (Berliner, 2002, p. 18).

Berliner names three reasons why educational research is so difficult. Firstly, the strong and uncontrollable influences which emanate from the contextual conditions of the research situation and which practically exclude a generalisation of the achieved results in the form of universal principles or covering laws. Secondly, myriads of interactions of the n-th order between the variables of a study which push the researcher into a wilderness of mirrors. And thirdly, the short half-life of educational knowledge due to the historic relativity and the social character of educational phenomena. These three characteristics of educational matter make educational science 'the hardest science of all' (p.18).

However, this does not exclude the experimental method from educational research. One merely has to be aware that researching a complex issue with the analytical methods of science is based on an idealisation. This idealisation is based on treating a complex matter *as if* it were complicated. Only under these circumstances the strict criteria demanded by the experimental method can be met. Indeed, the physicist Hans-Peter Dürr (1995), who recently passed away, argued that the approach used by the natural sciences relies on reducing complexity treating it approximately as complicatedness.

Adopting this point of view one can argue that the complexity of educational matters is not inaccessible to experimental research as it is advocated by the evidence movement. One only has to be aware of the idealisation that is assumed by using this method. For the knowledge base of educational practice, however, this means that the 'evidence' which science provides is of limited use for this practice. While the researcher may treat a complex phenomenon as if it were complicated, the practitioner rarely is confronted with this option. (S)he has to consider conditions which the experiment excludes by design: multicausality, interactions between conditional factors, non-linear relationships, feedback loops, dynamic processes which constantly change the causal structures, etc. Teachers are confronted with events which are difficult to foresee, which often accumulate, hardly ever leave time for in depth reflection and demand immediate reaction (see Herzog, 1999; 2002, p. 419ff.). Complexity also entails that no situation is exactly like any other.

A Short Outlook

We have now found the systematic argument which speaks against basing teaching on evidence in Slavin's sense. The idea that professional educational practice could be withdrawn from the necessity of subjective decisions and be based on evidence gained from randomised experiments is simply absurd. Science, as it is presented to us now, is without doubt helpful for understanding educational practice, but only of limited use if the aim is guiding or prescribing this practice. And this is not the case because educational research can be accused of being backward and therefore being responsible of the practitioners' knowledge deficit, but rather because this deficit is the result of a *constitutive* difference between the research situation and the practice situation.

By reverting to Shulman we have linked this difference to the concept of complexity. In the words of Berliner, we could also talk about contextuality. In the end, both mean the same – once from the scientific and once from the practical perspective. Shulman rightly differentiates two kinds of complexity: social and contentual complexity:

> […] all teaching – even the ostensibly simple teaching of arithmetic – is incredibly complex and enormously demanding. […] It is not only the multiplicity of roles that we, as teachers, have to play […] that makes teaching complicated. The pedagogy of subject matter for understanding is both a handful and a mind-full all by itself (Shulman, 2004, p. 512f.).

In one instance we deal with social complexity – because teaching usually takes place in a class situation. In the other instance we must deal with contentual complexity – because the relationship between teaching and learning is not a logical but a contingent one. 'We thus encounter two sources of complexity: the intellectual demands of deep disciplinary understanding paired with the social demands of coping with the unpredictability that accompanies such teaching (p. 513).

Both forms of complexity – the social as well as the contentual – play a key role in the teaching of mathematics, but while the social complexity is not specific to mathematics, the contentual is.

As far as I understand ALM's activities it seems to me that this is even the focus of your organisation. By focusing on adults it becomes more than clear that the abstractness and universality of mathematics present specific challenges for its learning. And while I do not think that these challenges are only specific for adults, they become clearer in this context – as if examined with a magnifying glass. Engaging with adults' mathematical competences does not least of all mean to become aware of the contextuality of using mathematics. Competences stand for the ability to act, that is, they are ideally useful out of school – in 'real life', where one cannot help but take the context into account. Every

situation in which mathematics is applied is unique in its own way. Mathematical knowledge can therefore never simply be applied, but has to be adapted to the specific situation.

Mathematics is therefore integrated into daily actions to the same extent to which knowledge about teaching is only of practical use if it is anchored in the teacher's actions. The relationship between theory and practice is therefore comparable in the case of mathematics itself and its teachings. Shulman goes back to Aristotle in order to characterise this relationship between theory and practice: '[…] theories are about *essence*, practice is about *accident*, and the only way to get from there to here is via the exercise of *judgment*' (Shulman, 2004, p. 534).

There is no mechanical link between theory and practice. One cannot do without the power of judgement: 'The process of judgment intervenes between knowledge and application. Human judgment creates bridges between the universal terms of theory and the gritty particularities of situated practice (Shulman, 2004, p. 534).

This reminds us of William James: *his* name for human judgement was an intermediary inventive mind.

My question which heads this presentation has therefore to be answered with *no* – provided that evidence is understood in the sense of Slavin and that of other representatives of the evidence movement, namely orienting educational practice along a technological understanding of science. However, if we understand evidence rather in its everyday sense, then an evidence base is also possible for the teaching of mathematics.

But we would be better advised to call it a scientific base or a knowledge base, in order to avoid misunderstandings. A knowledge base cannot be used technologically – as we have seen in James and Gage and how it was confirmed by Shulman. It presupposes intuition, creativity, talent for improvisation and judgement. Humans stand between knowledge and its application – in the form of a teacher in general or in the form of a mathematics teacher more specifically.

In my opinion this makes a teacher's job much more interesting than if it were reduced to that of an implementing body of educational research, research which aims at dictating to the last detail what works and what not and how the teacher has to behave in the classroom.

As you can see, I have managed to come full circle and returned to mathematics. I apologise for not drawing a bigger circle, as I am a mere educational psychologist. But I do hope that this presentation has provided a useful framework for your discussions in the coming days and therefore laid a fruitful ground for all the others to come.

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Coordinating Learning Inside and Outside the Classroom in Vocational Education and Training (Vet)

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Abstract

For more than 100 years now teachers have been complaining that their new learners 'cannot calculate anymore (!)'. As it is very improbable that the situation has deteriorated from generation to generation over the last hundred years, we at the Swiss Federal Institute for Vocational Education and Training (SFIVET) thinks that there must be something wrong with the teachers' expectations. As a consequence we started the project 'Everyday mathematics at work'. The idea is to bring the two learning sites – school and company – closer together.

Keywords: vocational, work-based, apprenticeship, training

Introduction

In Switzerland two thirds of adolescents start vocational education and training (VET) after lower-secondary education, i.e. after their compulsory nine years of schooling. VET is predominantly based on a dual system: practical, work-based training (apprenticeship) on three to four days a week at a host company is supplemented by theoretical classes (vocational subjects and subjects falling under Language, Communication and Society, LCS) on one to two days at a VET school. Vocational Subjects are usually not split up in separate 'subjects' but taught in a holistic manner.
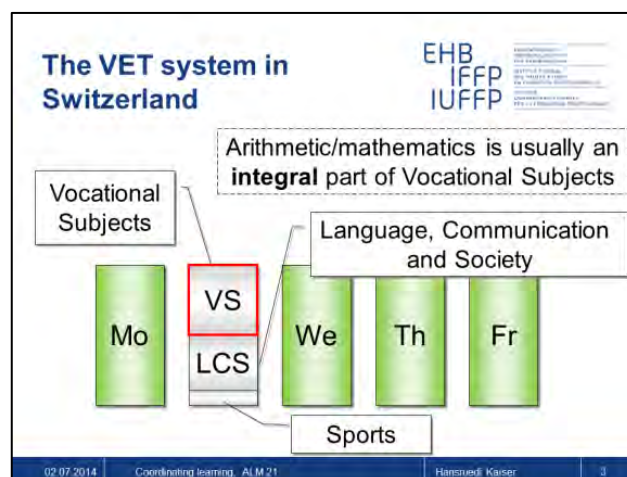

Figure 1: Four days at work, one day at school

For details see: http://swisseducation.educa.ch/en/vocational-education-and-training-0

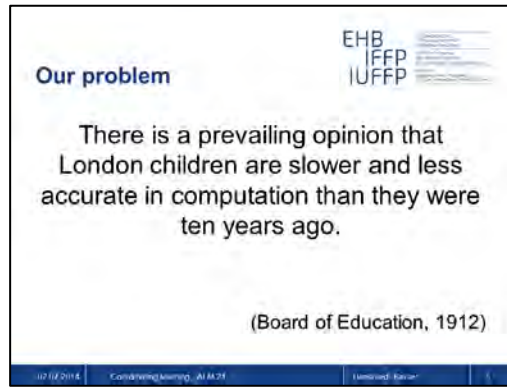## The project 'Everyday Mathematics on the Job'

Figure 2: VET teachers complaining then (and now)

For more than 100 years now teachers have been complaining that their new learners 'cannot calculate anymore (!)'. As it is very improbable that the situation has deteriorated from generation to generation over the last hundred years, we at the Swiss Federal Institute for Vocational Education and Training (SFIVET) think that there must be something wrong with the teachers' expectations.
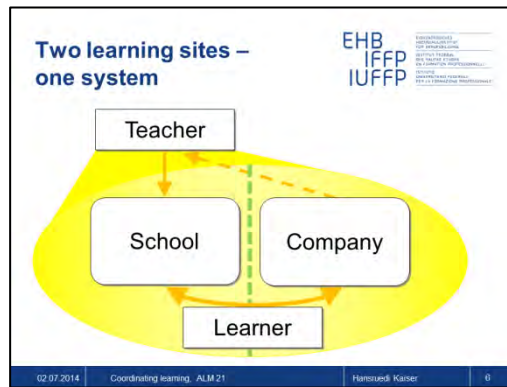

Figure 3: Helping learners by integrating school and work

As a consequence we started the project 'Everyday mathematics at work'. The idea is to bring the two learning sites – school and company – closer together so that the learners do not have to 'cross boundaries' every time they go from school to work or vice versa, but experience the VET system as an integrated whole. One favourable precondition to this is that the teachers themselves are experienced professionals in the occupation they are teaching. They know both worlds very well and are in a good position to help the learners to integrate their learning at the different learning sites.

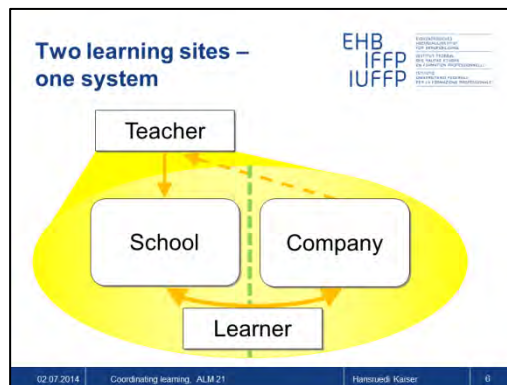## Background theory: Situated cognition

Figure 4: Stories about typical situations in everyday work life used as unifying language

To integrate the two learning sites the people at the two sites have to find a common language to talk about what the learners should learn. Abstract descriptions of competencies as found in regulatory frameworks are not very helpful, as they are insufficient to describe the ambiguous and open-ended challenges at the workplace (see e.g. Coben & Weeks, 2014; Lum, 2004). We found it more suitable to describe these challenges as typical situations of everyday life at the workplace (Kaiser, 2005a). To describe the situations we use stories which enable us to capture 'soft' aspects that otherwise are easily lost. So the unit of teaching is always an authentic situation and the challenges it poses.

Figure 5: 100% being 'the whole' does not work when baking bread

This connects directly with the situatedness of knowledge. We believe that the complaints of the teachers about learners who 'cannot calculate anymore' are a direct consequence of them not seeing this situatedness. E.g. children usually learn to use percentages in a context where it is natural to think of 100% being 'the whole' (Figure 6; left side). If all went well they can handle all kinds of situations where this idea of '100% is the whole' is applicable. However, when they enter an apprenticeship as baker they encounter professional bread recipes (Figure 6; right side). In this context, all ingredients are specified in percent of the amount of flour – which in no intuitive way is 'a whole'! Intuitive 'wholes' would be the dough mixed out of all ingredients or the finished bread. Searching for an intuitive 'whole' the learners stumble and the teachers – rightly – complain that they 'cannot do percentages'.
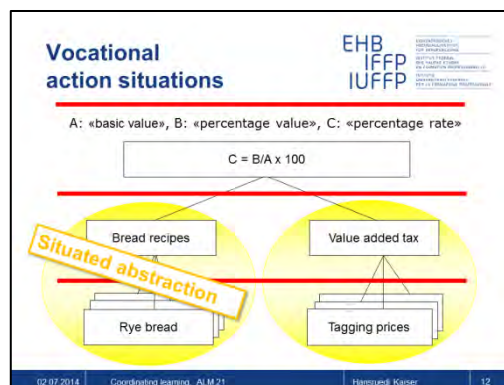

Figure 6: The two situations where bakers have to deal with the % sign

As a consequence we tell our teachers that they should not try to teach their learners 'percentages' (or 'the rule of three', etc.) but teach them to make bread. If the learners learn to use 'baker percentages' in the situation of 'making bread' (situated abstraction, Hoyles & Noss, 2004) and do not realize that, from a more abstract mathematical point of view, the 'baker percentages' are the same as the 'part of a whole percentages' they will still be good bakers. All the professions we have worked with so far do not know many different professional situations where the same mathematical concept is applicable. For example, the only other situation where bakers have to handle percentages is when they 'calculate VAT'. It is therefore no problem to treat them as two different situations at school – and they are

different: While calculating VAT the exact percentages to several decimal places is important; this is not the case with the percentages in a bread recipe.
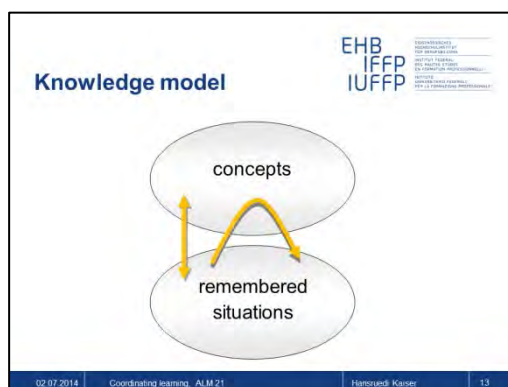

Figure 7: Two types of knowledge

To explain to our teachers why learners think as they think, we use a model which works with two different types of knowledge: Memories of self-experienced situations and learned concepts (Kaiser, 2005b; see Vergnaud, 1990 for a similar conception). Every time a learner encounters a new situation he or she is reminded of previous similar situations and tries to deal with the new situation in analogous ways as in these pervious situations. As a consequence, the learners' knowledge is portioned in packages of similar situations. Some learners manage to fuse the two packages 'part of a whole percentages' and 'baker percentages' to one package, but many do not.

Didactics: Learning how to use mathematics

The didactical model consists essentially of two rules:

1. Work from the concrete application to the abstract rules – and not the other way round.
2. Always stay in the context of situations that learners know from their workplace.
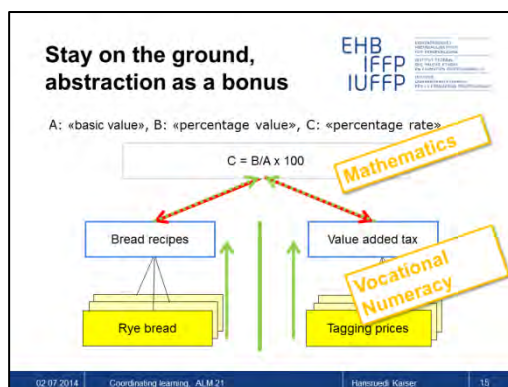
Figure 8: Rule A: Teach 'to make bread' and not 'percentages'

Rule A was mentioned before: 'Do not try to teach your learners 'percentages' but teach them to make bread'. This means: Do not start by recapitulating 'percentages' and then apply them to bread recipes. Start with bread recipes and explain, what the % sign means in this context, and make sure that the learners learn to handle the situation 'make bread' rather than 'calculating percentages'. Be prepared when later switching to the VAT to start again at the bottom and to explain, how the % sign is used in this new context. Important is that the learners learn to handle each of the two situations professionally (cf. 'contextual coherence', FitzSimons, 2014). Not so important is that they see the 'mathematical' similarities between the two situations ('conceptual coherence', FitzSimons, 2014).

Of course, if there is time and the learners are motivated, it is a good idea to discuss with them later on – after they feel confident with both situations – the similarities between the two situations and do some 'mathematics'. This will help them later on to adapt to changes at the workplace or to continue with a program in higher education. But you will not be able to do that with all of your learners.
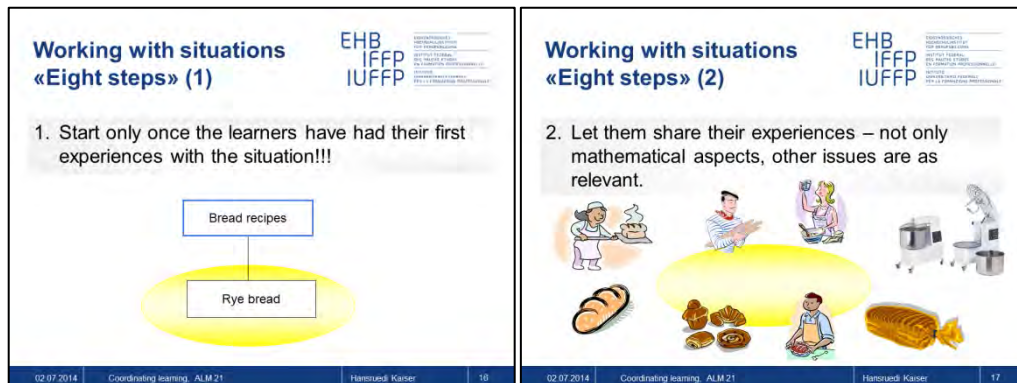

Figure 9: Bring the situation to the classroom (Steps 1 and 2)

For rule B we propose an eight step didactical model to our teachers. Step 1 in this model is actually more a stop-sign then a step. It just means: Do not try to teach the learners how to handle a situation they have not yet experienced at their workplace; it is a waste of time (see LaCroix, 2014)!

To work with a situation like 'baking bread' it is important that teachers and learners activate as many remembered situations as possible. This is the idea behind step 2. By listening to the learners' stories the teacher also gains insight into how the learners perceive the situation and how this perception possibly differs from his professional perception.
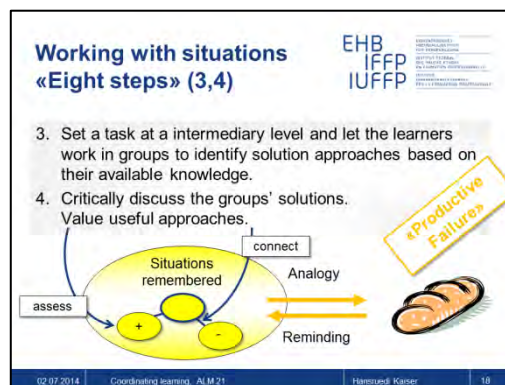

Figure 10: Find out and value what they already know (Steps 3 and 4)

Step 3 and 4: Kapur & Bielaczyc (2012) explored this way of working with learners' prior knowledge under the heading of 'productive failure' and showed how effective it can be. The idea is to start with what the learners already know instead of complaining about what they do not know. Working on the task and discussing the solutions has two functions: 1) Connecting what will follow with the already existing experiences, 2) critically evaluating these experiences in the light of a professional way of handling the task. These old experiences will stay in the package of similar situations and will continue to influence what the learners do when baking bread. So it is important that the learners know which remembered situations are reliably good examples and which are examples of situations to avoid. This is possible when the task is simple and familiar enough to remind them of earlier experiences and at the same time demanding enough so that they encounter the limits of their prior knowledge (productive failure).

Ideally, a list of open questions is the result of step 4; questions on which the learners agree that they need an answer for. Sometimes there are no questions because the learners handled the situation already perfectly well. In this case the rest of the steps can (and should!) be skipped.
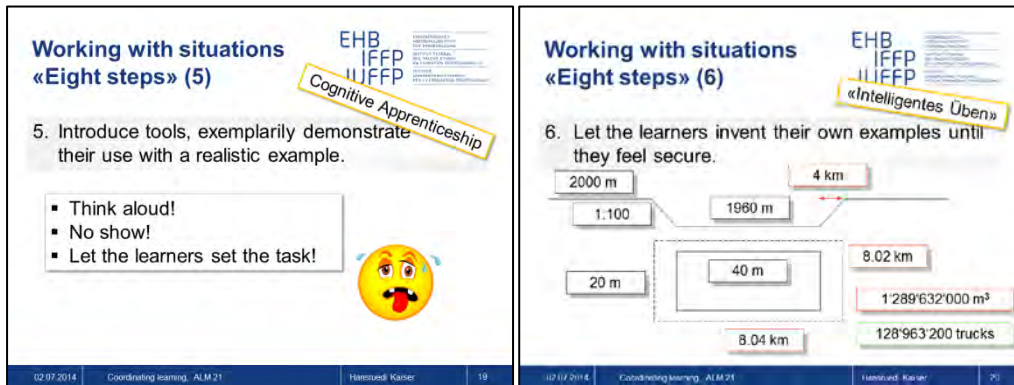

Figure 11: Model a professional solution and let them practice (Steps 5 and 6)

Step 5 provides the answers to the open questions from step 4 in form of a demonstration of how this type of situation is professionally handled. It corresponds to the modelling-step of the Cognitive Apprenticeship process (Collins et al., 1989; Weeks et al., 2013). We always tell our teachers that they should provide a real model and not a show. The learners should see and hear what a professional thinks and where even a professional has to think hard. As a rule we propose to not prepare a demonstration but to let the learners set the task and then try to solve it in front of them while thinking aloud.

Step 6 corresponds to the 'coaching', 'scaffolding' and 'articulation' parts of the Cognitive Apprenticeship process. Details about what is important in this step can be found in publications about Cognitive Apprenticeship. As an addition we propose our teachers not to work with a list of prepared examples but to let the learners invent their own examples ('Intelligentes Üben' [Intelligent practice], Leuders, 2009). There are several advantages to this: First, you do not have to prepare anything! Second, learners usually find teacher set tasks boring, but enjoy working on tasks prepared by their colleagues. And third, learner constructed tasks sometimes explore aspects of the situation a teacher would never think of. My favourite example is from the time when I worked with construction workers. The task was to calculate how many truckloads of dirt had to be removed while excavating a pit. They decided to make a deep pit (40 m), with walls not too steep (1:100) to reduce the risk of a collapse. That gave them an upper rim of the pit of 8 by 8 kilometres and about 129 million truckloads of dirt. We laughed a lot but at the end several of the construction workers said that the example helped them a lot to understand what a slope of 1:100 or 3:4 really means. With the usual teacher set examples with 'realistic' slopes of 2:1 und 3:2 there would not have been enough variation to get a feeling for the differences.
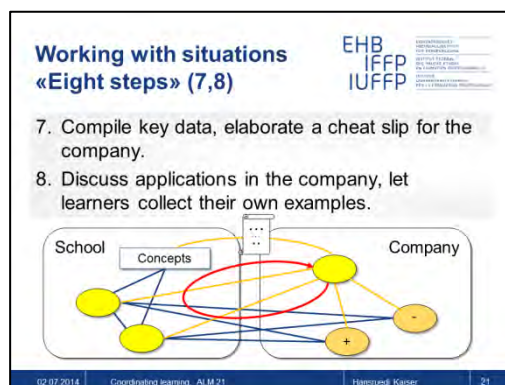

Figure 12: Help them to transfer to the workplace (Steps 7 and 8)

The function of the last two steps is to bring the learning process from the classroom back to the workplace. Step 7 prepares that move. The idea is that the learners construct an external memory that will help them to remember essential details of what they learned in school, once they are back at work. Step 8 has two parts. Part one is a discussion where the teacher and the learners try to anticipate what will happen when the learners begin to use at the workplace what they just learned in school. Part two takes place a week (or more) later. The learners come back to the classroom, tell what has happened, what did work and what did not, and where the problems were when they tried to apply the concepts and techniques learned at school. Solutions for these problems are discussed together and ideally at the end – after several weeks – every learner can add at least one positive example to his or her memory of remembered situations.

If all goes well, what happens by following the 'Eight Steps' is: The learners start with some remembered situations from the workplace (the brown circles in Figure 12). In steps 3 and 4 they learn in which instances these experiences have already been helpful to solve a new task and in which they have not (the plus and minus signs within the brown circles). Then in step 5 and 6 they add a few new situations to their memory by watching the teacher model and by working on self-constructed tasks. These memories are connected to the old situations and to some theoretical concepts (blue lines). Before going back to work they write a cheat slip (a kind of boundary object; Hoyles & Noss, 2004) which is in their memory also connected to the school situations (brown arc on top). Back at work they encounter a new situation (yellow). This new situation will remind them of the old workplace situations, which will remind them of the new school situations, which will remind them of the newly learned concepts and the cheat slip. Based on all the remembered situations, the new concepts, and the cheat slip, they will try to handle the new workplace situation. They will end up with a new (hopefully positive) memory of a workplace situation which is not only connected to memories of old workplace situations but also to memories of school situations (red arc).
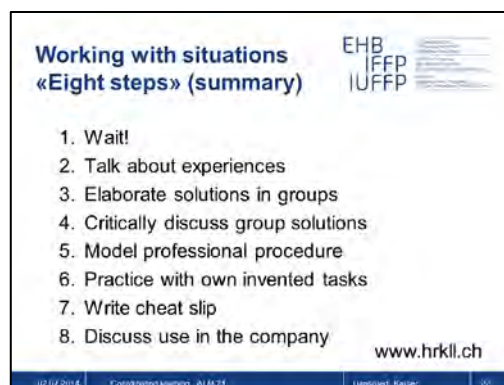

Figure 13: The 'Eight Steps'

I presented the 'Eight Steps' here in the way we tell our teacher trainers what they should tell the teachers about what the teachers should do in their classrooms so that in the end the learners learn something useful for their work at the workplace.
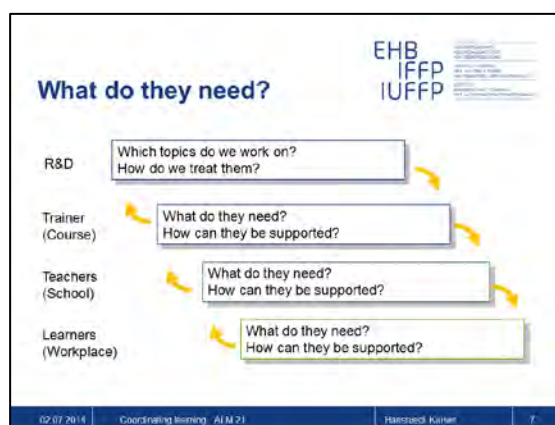
Figure 14: The transmission pipeline

Conclusion

There are many details to each of the eight steps and it is not very likely that each and every one survives the transmission pipeline (see Figure 14). When we watch teachers we see many 'mutations' to our ideas – even 'lethal' ones (Brown & Campione, 1996). But one advice seems to survive: 'Do not prepare 'word problems'; work with real situations the learners tell you about'. Already, this is great, because once the teachers start to do this, they have to do steps 1 & 2. They will then realize that the learners know more than they always thought (steps 3 & 4) and they will have to tune their 'model' (step 5) to what is really going on at the workplace. This will help them realizing that applying this 'model' at the workplace (step 8) is never straightforward but a major step. All this happens because once they allow the learners to talk about what is going on at the workplace, the learners will insist on making connections between school and work. As two teachers told us: 'The learners start to feel co-responsible for what is going on in the classroom. They want to show us how it is really done at the workplace. And they become co-teachers explaining and showing things themselves to each other.' (For more information about the experiences of the two teachers, see Califano & Caloro, 2013.)

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Section 1.b.

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Maths Everywhere – An Interactive Learning Tool for Everyday Life

Susan Easton

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The GO Model: Swiss Model to Promote Basic Skills in the Workplace

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Mathematics in Multilingual Classrooms: From Understanding the Problem to Exploring Possible Solutions

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Interpretations of the problem

- Its about the learners
  - *Who they are? And why they are learning mathematics*
- Its about mathematics itself
  - *Learning mathematics is similar to learning a language*
  - *Mathematics functions as both medium and message (Pimm, 1987)*
- Its about language
  - *But solving the language fluency issues alone will not solve the problems*
- Its about the quality of mathematics teaching and learning in multilingual classrooms

It is about all these:
Learners, mathematics, language and pedagogy!

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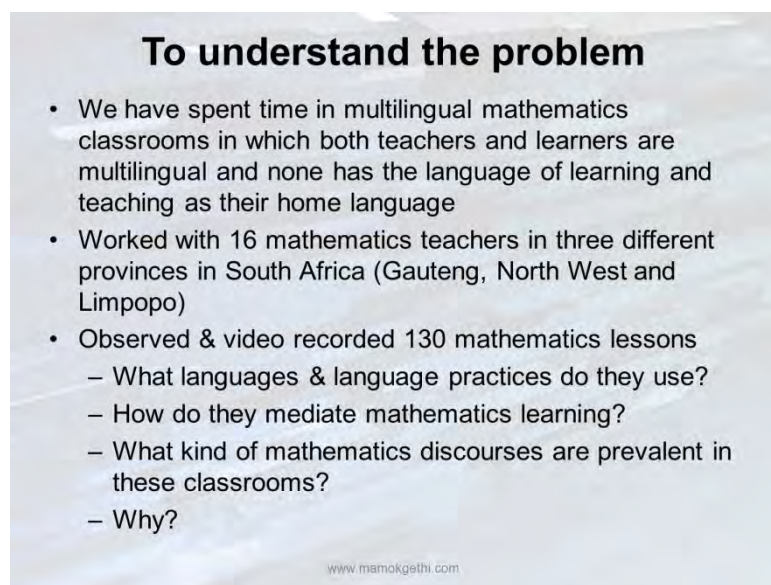
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Central to the problem is a need to address the uneven distribution of knowledge and success in mathematics.

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To understand the problem

- We have spent time in multilingual mathematics classrooms in which both teachers and learners are multilingual and none has the language of learning and teaching as their home language
- Worked with 16 mathematics teachers in three different provinces in South Africa (Gauteng, North West and Limpopo)
- Observed & video recorded 130 mathematics lessons
  - What languages & language practices do they use?
  - How do they mediate mathematics learning?
  - What kind of mathematics discourses are prevalent in these classrooms?
  - Why?

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What we found

- Dominance of English despite learners' limited fluency in it, accompanied by the prevalence of procedural discourse:
  - conversations, actions and behaviours focusing on the procedural steps to be taken to solve mathematics problems
- Limited occurrences of code-switching, accompanied by conceptual discourse:
  - conversations in which the reasons for calculating in particular ways and using particular procedures to solve a maths problem also become explicit topics of focus

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What we found

- Learners exposed to low cognitive demands mathematics tasks that also seem to have no purpose. For e.g.
  a) In the SPCA are 12 cages; in each cage are 12 dogs. How many dogs are there altogether?
  b) Solve for x in $2x + 5 = 8$

… and more questions emerged

What shapes the nature of the mathematics taught, the mathematics tasks used and the language choices made in these classrooms?

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What we found

- Teachers do not want to give cognitively demanding tasks that are wordy because learners are not fluent in the LoLT
- Teachers and learners prefer that English be used as the language of learning and teaching mathematics because of the hegemony of English and the desire/need gain access to social goods e.g. higher education; jobs

Decisions about which language to use, how, and for what in multilingual mathematics classrooms are not just pedagogic but also political

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Some theoretical underpinnings

- An understanding of language as a resource (see for e.g. Adler; Barwell; Gorgorio & Planas; Khisty; Moschkovich; Secada; Setati, Staats)

A different conception of language as a resource

- For a resource to be useful it needs to be both visible and invisible (Lave and Wenger, 1991).
  - *Visibility* is in its presence and the form of extended access to mathematics it provides
  - *Invisibility* is in the form of unproblematic interpretation and integration of language(s) used
  - For e.g., the use of technology in mathematics teaching and learning.

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Towards a possible solution

A teaching strategy guided by two main principles:
1. the *deliberate*, *strategic* and *proactive* use of the learners' home languages.
   - unlike code-switching, which is spontaneous and reactive.
   - English and the learners' home languages operating together and not in opposition.
   - tasks given to learners in multiple languages.
   - learners are explicitly encouraged to interact in any language they feel comfortable with.
2. the use of relevant and challenging mathematical tasks,
   - Through this, learners would develop a different orientation towards mathematics and would be more motivated to study and use it.

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What this solution is <u>NOT</u> about

- Developing the mathematics register or terminology in African languages
  - It is about using language as a transparent resource to make mathematics accessible to multilingual learners
- Developing a glossary of terms in African languages
  - It is about comprehension rather than just terminology.
- Developing learners' fluency in English or their home languages
  - While this may happen, it is not the focus
  - It is about developing the learners' mathematical proficiency
- Teaching solely in African languages
  - While this may be desirable it is not feasible at this stage given the demand for access to English

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Change in the classroom discourse

1. T: Okay, if you use electricity ukho bhadala forty rand *[Okay, if you use electricity will you pay forty rand]*?
2. Ls: Yes meneer *[Yes sir]*.
3. T: If unga shumisanga electricity ukho bhadala forty rand *[If you did not consume electricity, will you pay forty rand]*?
4. Sipho: No, no no …
5. Given: Haena, whether ushumisile ore haushumisanga, ukhobhadala forty rhanda *[No, whether you have consumed electricity or not, you pay the forty rand]*.
6. T: Whether ushumisile ore haushumisanga? *[Whether consumed or not?]*.
7. Sipho: Eya, yes, it is a must.
8. T: It is a must?

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Given: Hei, nayo…ah…(Giggles)…So forty rhanda hi monthly cost ne, then ba yieda nga twenty cents kha kilowatt for one hour. Then after that, angado shumisa…baibidza mini? Heyi … ndoshumisa one kilowatt nga twenty cents kha one hour *[Hei, this question … ah …(giggles) … So forty rand is the monthly cost, then they add twenty cents per kilowatt-hour. …, they use…, what do they call it? Heyi … they use one kilowatt-hour for twenty cents]*.
Sipho: Eya *[Yes]*.
Given: Boyieda, maybe boshumisa twenty cents nga one hour *[They add it, maybe they use twenty cents per hour]*.
Sipho: Eya, yantha *[Yes, one hour]*.
Given: Iba … *[It becomes…]*.
Given and Sipho: Forty rand twenty cents.
Sipho: Yes, vhoibadela monthly, ngangwedzi ya hona. Yo fhelela, yes. Sesiyaqubheka. *[Yes, they pay it monthly, each month. It is complete, yes. We continue]*.

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Learners' reflections on the lessons

Interviewer: I understand this week you had visitors in your class, what was happening?
Sindiswa: Er…, we were learning a lesson in which we can calculate electricity er …. amount … er … the way in which the electricity department can calculate the amount of electricity unit per household.

Nhlanhla: We were learning about how to calculate …er…er… kilowatts of the electricity, how do we … like … how can we calculate them and when … at …, Besifunda mem ukuthi ugesi udleka kakhulu nini.
[We were learning about when there is high electricity consumption.]

Colbert: Er …we were just solving for electricity, kilowatt per hour, for comparing if they are using card or the meter, which is both, I think are the same.

Sipho: Er, the visitors they were doing research.

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Interviewer: What ... what was so special about the lessons?

Sindiswa: It does not include those maths ... maths. It is not different, but those words used in Maths didn't occur, didn't occur but we weren't using them. ... Er ... 'simplifying', 'finding the formulas', 'similarities', ...

Nhlanhla: Hayi, no mem, ku-different... Okokuqala mem, ilokhuza, la sidila ngama-calculations awemali, manje ku-maths asisebenzi ngemali.

[No mam, it is different. Firstly mam, we were working with money and usually in maths we do not work with money.]

Colbert: Iya, basenzele in order to ... ukuthi ibe simple and easy to us, because most of people, uyabona, aba-understendi like i ... like i-card ne meter. Abanye bathi i-meter is ... i-price yakhona i-much uyabona, i-card iless i-price yakhona, that's why uyabona. So, abantu abana-knowledge, uyabona, bakhuluma just for the sake of it. So, I think for us, because we have learnt something, both are the same.

[yes, you see they made it easy for us, because most people do not understand, like card or using a meter. Some say when using the card you pay less than when using the meter, you see. So people do not have knowledge out there, they just talk for the sake of it. So think, for us we have learnt something, both are the same.]

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Interviewer: So what is it that you like about the new approach that Mr Molefe was using?

Sindiswa: Ke gore, the way ne diquestion di ne di botswa ka teng, it was easy for the whole class for all of us, for all the students to understand and answer all the questions.

[The way the questions were asked, it was easy for the whole class, for all of us, for all students to understand and answer all the questions.]

Nhlanhla: I think iyasebenza, ngoba ama-learners amaningi, maybe like, uma ungasebenzise ama-home language wabo, abaphathisipheiti kakhulu. Mabanikezwa ama-home language abo, I think bazokhona ukuphathisipheita.

[I think it works because many learners, maybe like, when their home language are not used they do not participate. When they are given their home languages, then they are able to participate.]

Colbert: Because most of us we are ... be baphathisipheitha. The whole class, I think be-iphathisipheitha. But before beyiyenza ukuthi like, beyi ... bebabona nje i-class, kukuthishwa maybe four learners uyabona, others ... (Inaudible).

[Because most of us we are...we were participating. The whole class I think was participating. But before then, it was as if the teacher is talking to only four learners, others ...]

Sipho: Because kaofela digroup they were participating, wa utlwisisa mam. Le bane ba sa phathisipheiti ko klaseng, ne setse ba phathisipheita. Nna ke maketse gore 'he banna, mothaka o kajeko ke ena oe arabang so Maths' (Clicking fingers).

[Because all the groups were participating, you understand mam. Even those who never participate were participating. I was surprised that 'hey man, even this guy is answering questions today in maths?']

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What the analysis shows?

- This approach creates an opportunity for learners to engage with higher cognitive level demand mathematics tasks.
  - There is a purpose to the mathematics task
- Dominance of conceptual talk during lessons
  - No pressure to come up with an equation, focus is on solving the real problem.
- Language functioning as a transparent resource (visible and invisible)
  - No restriction on which language to use, focus is on the mathematics task
- Learner participation and interest in mathematics.

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What we can say about the strategy at this stage?
- Potential to increase learner participation and interest in mathematics
- Engagement with higher cognitive level demand mathematics tasks
- Language as a transparent resource

What we know?
- There is no single, universal teaching strategy that suits all learners, situations and contexts
- Different kinds of multilingual classrooms ay need different strategies.

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In short
What difference is this solution making?

- Learners' interest in mathematics
  - **From** the teacher interacting only with a few in class **TO** all learners being engaged with the task
- The nature of mathematics tasks
  - **From** low cognitive demand meaningless tasks **TO** tasks that draw on the learners' environment as a context for instruction
- Use of languages
  - **From** choosing one language for learning **TO** drawing on multiple languages during learning
- Mathematics discourses
  - **From** abbreviated one word responses **TO** explanations of how particular problems are solved and why

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Key theoretical shifts in the field

- From bilingualism to multilingualism - a global phenomenon, which until the nineties was not taken into consideration by research in mathematics education (Adler, 1995; 1997; Setati, 1998)
- From constructing language diversity as a problem to constructing it as a resource (Setati, 1998)
- From a conception of language only as a tool for thinking and communication to a recognition of the political role of language and how that shapes mathematics teaching and learning in linguistically diverse classroom as well as research in this area of study (Setati, 2005).
  - language choices in linguistically diverse classrooms are also shaped by the economic and socio-political context in which learning takes place.

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SECTION 2

Presenter Contributions

Section 2.a.

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Making Maths Useful: How Two Teachers Prepare Adult Learners to Apply Their Numeracy Skills in Their Lives Outside the Classroom *

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Abstract

This pilot case study of two teachers and their learner groups from Adult and Community settings, investigates how numeracy teachers, working with adult learners in discrete numeracy classes, motivate and enable learners to build on their informal skills and apply new learning to their own real-life contexts. Teachers used a range of abstract and contextualised activities to achieve this. Similarities and differences between teachers' approaches were analysed using a Context Continuum model. Whether teachers started with real-life situations then moved to the abstract mathematics within them, or approached it the other way around seemed less important than ensuring there was movement back and/or forth between the different discourses of numeracy and mathematics.

Keywords: context continuum, numeracy, out-of-school practices

Introduction

The inherent complexities of developing an adult learner's numeracy knowledge and skills in a way that will both enable them to pass a summative assessment in order to gain a qualification, as well as develop the motivation and ability to 'transfer' and use their skills and knowledge to support their own real-life problem solving, are widely debated, and relevant internationally. This research investigates whether and how numeracy teachers of adult learners enable learners to apply their skills to real-life uses, particularly in 'discrete' numeracy classes, i.e. those which are not vocationally or workplace-based.

Mathematics and numeracy qualifications in the Further Education (FE) & Skills sector in the UK identify the contextualised and embedded agendas within which adult numeracy teachers are working:

> Functional skills are the fundamental, applied skills in English, mathematics, and information and communication technology (ICT) which help people to gain the most from life, learning and work.

(Ofqual[1], 2012)

Prior to the relatively recent introduction of the Functional Mathematics curriculum, the preceding Adult Numeracy core curriculum (BSA, 2001) stated that it is deliberately context free so that numeracy teachers can relate the curriculum to their learners' own contexts.

The stated intentions of helping people to gain the most from life, learning and work, and relating the curriculum to learners' own contexts, raise some interesting questions; for example, are these intentions consistent with learners' and teachers' intentions and aims? Research (Coben et al., 2007; Swain, Baker, Holder, Newmarch, & Coben, 2005; Swain & Swan, 2007) suggests that many learners simply want to gain a qualification to enable them to gain access to other programmes of study or to

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[1] Ofqual – the Office of Qualification and Examinations Regulation.

enhance their job prospects, other learners wish to be able to help their children. In summary (Swain et al., 2005) explain that:

Students' motivations are varied and complex but few come to study maths because they feel they lack skills in their everyday lives.

There are also difficulties associated with teaching numeracy and mathematics in a way that enables learners to apply what they learn outside the classroom, partly because of the differences between the approaches used to problem solve inside the classroom - to ultimately enable learners to gain a numeracy qualification - and the approaches we use to problem solve in real life (Ivaniĉ, Appleby, Hodge, Tusting & Barton, 2006), further amplified if the summative assessment is not fully aligned with the intended outcomes. These approaches are sometimes respectively referred to as 'school maths' and 'street maths' (Nunes, Schliemann, and Carraher, 1993). Therefore teachers need to make choices about the extent to which they balance these different approaches or types of numeracy (Kanes, 2002), and the methods they use to aid learning.

Other considerations must be where and how numeracy is likely to be used in learners' lives in terms of citizenship, learning, work, and life in general, and the extent to which it is possible to help learners to apply their knowledge of mathematics or numeracy to real-life scenarios, and to 'transfer' their skills to different situations (Lave, 1988). How successful are teachers in achieving this?

In summary, a number of underlying questions are raised:

1. What are teachers' aims for numeracy learners?

2. What are learners' aims? (Do their aims include learning maths in order to be able to apply it to their own real-life contexts?)

3. To what extent are teachers successful in enabling learners to apply the mathematics they learn?

4. What methods do teachers employ in order to help bridge the gap between abstract mathematics and useable numeracy?

This paper, based on a pilot case study of two numeracy teachers and their learner groups, gives a brief summary of the findings of the first two questions, but focusses mainly on questions three and four. During the data analysis stage I developed a 'Context Continuum' model which might be of use to teachers, teacher educators and possibly researchers in providing a means of discussing the extent to which different teaching and learning activities are embedded into real life contexts, and to discuss ways in which teachers can help learners to make links between the methods and processes of discrete mathematical concepts and applied problem-solving in real-life contexts.

Prior to outlining the study's methodology and findings, a review of relevant literature and research is presented to provide further background to the study, as this was used to inform the collection, and to some extent, the analysis of the research data.

Theoretical background

In particular, this section focuses on two main areas: firstly, the different kinds of numeracy that exist, and the difficulties and contradictions that this creates in terms of learning and teaching; and secondly, consideration of the adult numeracy teacher's role in facilitating learners to make sense of these different types of numeracy.

Different kinds of numeracy

Many research studies in the late 80's and 90's, for example, Lave (1988), Saxe (1988), Nunes, Schlieman and Carraher (1993), Harris (1991), and Hoyles, Noss and Pozzi (1999), investigated the numeracy practices that people used in life and in work, and examined the differences between the maths learning that takes place in work (often referred to as 'street' mathematics), and the maths learning that takes place in a more formal learning environment such as school (referred to as 'school' mathematics). Lave's research (1988) challenged the idea that mathematics can be taught in a formal setting and then that knowledge can be transferred and applied to a vocational area or to everyday

practice. Nunes, Schliemann and Carraher (1993) also suggest that the maths used in working practices is best learned within those practices, which supports the idea of 'embedded' learning being carried out in the vocational context rather than in the classroom. This research supports Barton's (2006, p. 13) social practice perspective in which he questions the idea of "numeracy as itemised, transferable skills" as he suggests that numeracy processes are not easily detachable from their context. However, in the UK not all numeracy learning takes place in vocational contexts, instead, discrete numeracy classes are available to adult learners.

One aspect explored in these studies is that formal and informal techniques use different mathematical practices, for example in street maths, there is often more emphasis on mental maths and estimation, whereas in school maths learners generally expect to use specific written algorithms to apply to problems that they do not see as real life scenarios. Such differences in approaches were borne out in Jurdak and Shahin's study (2001, as cited in FitzSimons, 2008) which explored the differences in the types and sequences of the actions of a group of five experienced plumbers and a group of five school-children in creating a cylindrical container (given a specific height and capacity) from a plane surface. The plumbers engaged with the physical resources available and used a kind of trial and error approach to refining their model whilst the students engaged mainly with cognitive tools to select the formula and calculate the unknown. Obviously these different approaches have implications for numeracy teaching and learning, particularly if the intention is for learners to be able to apply their learning to work and other real-life contexts.

Oughton's (2009) and Dowling's (1998) research is also consistent with these ideas. Dowling explores the difficulties presented in the linking of mathematics to real-life scenarios, in written mathematics school texts, highlighting the conflicts that result and the unrealistic scenarios that are consequently played out. He summarises:

> School mathematics may incorporate domestic settings in its textbooks, but the structure of the resulting tasks will prioritize mathematical rather than domestic principles. Alternatively, domestic practices may recruit mathematical resources, but the mathematical structure will be to a greater or lesser extent subordinated to the principles of the domestic activity.
>
> (Dowling, 1998, p. 24)

He is emphasising the important role that context plays in informing the approaches used in problem solving. In a maths classroom, a learner expects to use mathematics, whereas in a real-life problem-solving scenario, mathematics is but one factor. Oughton (2009, p. 27) supports the idea of unrealistic maths problems in classrooms, suggesting that often:

> students were required to willingly suspend disbelief where the narratives of word problems did not reflect the real world.

Clearly the careful selection and design of learning materials is important, if learners are to be able to make links between what they do in the classroom and how they can use their numeracy skills outside it. Such unrealistic problems have been identified as 'quasi' activities in the research study.

In fact, Kanes (2002) suggests there are three different kinds of numeracy, which he terms: visible-numeracy, constructible-numeracy, and useable-numeracy. Visible-numeracy is where mathematical language and symbols are used explicitly, usually in a learning environment; constructible-numeracy is where constructivism and social constructivism approaches enable learners to build on and transform previous knowledge adequately to problem solve; and useable-numeracy is where the specific numeracy tools and techniques used are "complex and deeply embedded in the context in which it acquires meaning" (Kanes, 2002, p. 4), for example, the workplace in which they are used. Building on the work of Noss (1998, as cited by Kanes, 2002), Kanes explains how the different kinds of numeracy might be considered to create tensions in designing a suitable curriculum, as they conflict with one another, for example concentration on visible-numeracy "oversimplifies issues relating to useable-numeracy, and this leads to numeracy becoming less "useable" than would otherwise be the case" (Kanes, 2002, p. 6). In designing a curriculum to meet the needs of all stakeholders, I propose that these tensions are at the core of curriculum planning for many teachers involved in numeracy teaching and learning.

Such challenges and tensions presented in teaching mathematics form the basis for Kelly's (2009, 2011) research, which was based around those teaching mathematics in a vocational context and which "highlight[s] the tensions between learning relevant mathematics skills in the workplace and those in education contexts" (Kelly, 2011, p. 37). She developed a model for conceptual analysis of these tensions, and the example in Figure 1 explores the contrasting approaches used in the classroom with those used in the workplace, e.g. using centimetres and metres in the construction classroom (presumably based on curricula requirements) whilst it is customary to use millimetres in the construction workplace. Likewise, in (UK) industry it is commonly known that 60 bricks will build $1m^2$ of wall, whereas in the classroom, this knowledge may not be used as the basis for calculations.

In seeking a conceptual model for my own research, Kelly's model (Figure 1) provides a useful starting point. In particular, the axis denoting Context (Work Life – Education) inspired 'The Context Continuum' model which I developed in order to support analysis of the extent to which contexts were embedded into different teaching activities, as identified in the collected data.
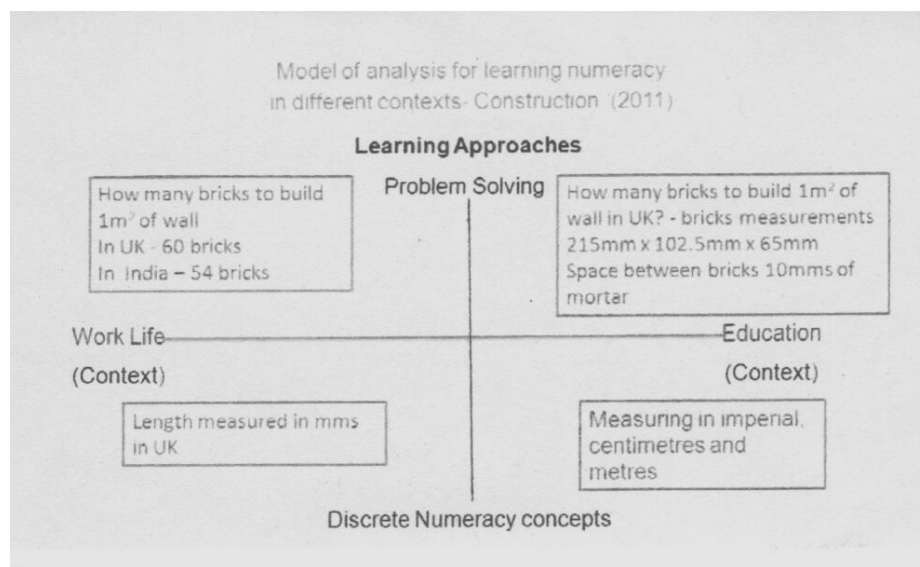
Figure 1. Kelly's model of analysis for learning numeracy in different contexts –
applied to construction (Kelly 2011, p. 41).

Having considered the different types of numeracy, the numeracy teachers' role in navigating through these is explored next.

The teacher's role and contextualisation

Because of the different types of numeracy that exist, the numeracy used by people on a regular basis, embedded into their daily contexts, is often 'invisible mathematics', a term used by Diana Coben (2000, p. 55) to describe "the mathematics one can do but which one does not recognise as mathematics". She goes on to explore the idea that the mathematics that people can do is often considered by them to be common sense rather than mathematics, which is consistent with learner 'Selena's' views in Swain et al.'s research (2005). Both the invisibility of the mathematics people already use and the status of mathematics in society means that mathematics is often seen by learners and others as "unattainable", something they "cannot do" (Coben, 2000, p. 55), and this impacts on a learner's self-confidence and also their perception of their intelligence or their ability to learn.

For some learners, making maths less abstract can help them make meaning, i.e. understand what it is they are doing (Swain et al., 2005). Contextualisation is one of the methods of meaning-making that Johnston (1995) explored, and it is also supported by learners in Fantinato's (2009) research, who were explicit about the fact that thinking in terms of bags of rice, beans or sugar rather than just numbers, makes things easier to learn. Conceptually difficult areas are often those which seem most abstract, for example negative numbers, which can be usefully related to credit and debt, in making sense of why, for example, 'two negatives make a positive'.

My belief is that in contextualising mathematics, teachers can also help make the 'invisible' visible to learners. If teachers are successful in making the maths more visible, by relating it to learners' life experiences, learners will no longer see maths as something they just do in a maths class, but they will see maths as a tool they can use to help them make informed choices and decisions about, for example, purchases, financial decisions and other contexts relevant to their lives. In addition learners may see that they already do some maths in their lives, therefore that they *can* do maths, albeit the informal 'street' maths. This can be used to build confidence and to help turn learners from an "I can't" to an "I can" kind of learner, which Marr, Helme and Tout (2003) explain as a shift in identity towards someone who is more numerate. Marr, Helme and Tout's (2003) model of numeracy competency, which was developed by a group of experienced adult literacy and numeracy practitioners in Australia, is shown in Figure 2 below:
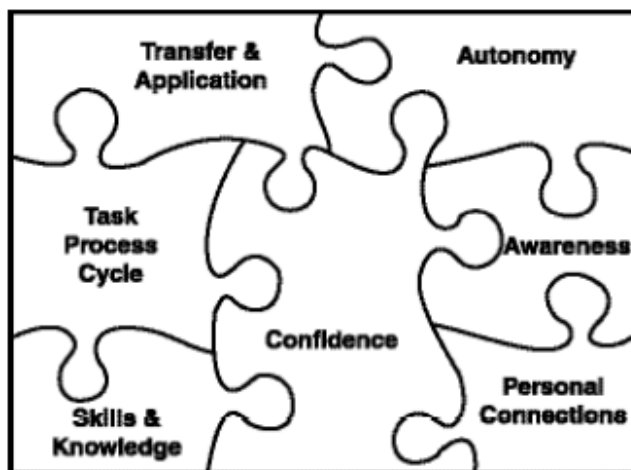
Figure 2. Model of holistic numeracy competence. (Marr, Helme & Tout, 2003, p. 4).

This model suggests that confidence is central, and perhaps the biggest single contributor, to a learner becoming competent in numeracy. The (cognitive) left hand side of the model considers different types and levels of skills and knowledge, which rise in complexity from the bottom to the top of the model, moving from abstract mathematical skills to numeracy skills applied in real-life situations, similar to Kanes' types of numeracy. The (intrapersonal) right hand side considers the building blocks which enable learners to become increasingly independent learners (from bottom to top), starting with linking learners' learning goals into their interests and motivations, and ending with someone able to take more control over their own learning.

Safford (2000, p. 6) identifies one of the roles of the maths teacher as being a mediator between 'street' maths and 'school' maths "to aid students in clarifying knowledge they already own, and to alter and enhance it with new knowledge acquired in our classrooms". Building on existing, informal skills is therefore a positive way to support learners to develop the kind of maths they need in the classroom. Saxe (1988, p. 20) suggests that otherwise:

> [the] processes of transfer are often protracted ones, ones in which [learners] increasingly specialize and adjust strategies formed in one context to deal adequately with problems that emerge in another.

Instances where the teacher has built on learners' existing, informal skills in the study have been identified as 'Validating'.

Safford and Saxe's suggestions are consistent with FitzSimons' (2006) research where she draws on Bernstein's work in discussing the need for teachers to help learners continually cross and re-cross "the borders of vertical discourse [of abstract mathematics] and horizontal discourse [of using numeracy in real contexts]" in the teaching and learning process "in order to develop the capacity for numerate activity" (p. 36). This suggests the need to move continually back and forth between formal,

abstract mathematics and informal real-life uses, in the numeracy classroom, and evidence of this was found in the study.

Summary

In summary, it is evident that varying contexts foster different approaches to, and priorities in the use of mathematics. In particular, the contrast between the way in which learners approach (often unrealistic) mathematics problems in the classroom, compared to the ways in which they approach the use of mathematics knowledge in real life contexts (including in the workplace), is so great that learners are often unaware of the mathematics they actually use in real life. This invisibility compounds learners' beliefs that they cannot 'do' mathematics, which affects their confidence and motivation.

Therefore the role of numeracy teachers is to cross and re-cross the different discourses of mathematics and numeracy within their teaching, despite the conflicts and challenges associated with this, in order to help learners acknowledge what they already know and use, relate it to the mathematics they do in the numeracy classroom, and build on their knowledge and understanding in a way which enables learners to use their new knowledge in their real-life contexts.

Having established the complexities involved in teaching and learning mathematics for the purpose of transferring and applying mathematical knowledge and skills to real-life contexts, the aim of this research is to identify whether and how teachers can enable learners to achieve this.

The study

Punch (2009, p. 119) explains "the case study aims to understand the case in depth, and in its natural setting, recognising its complexity and its context", therefore this is a very appropriate methodology to develop an understanding which included the perspectives of numeracy teachers and learners in the research. For the pilot case study two teachers were selected by purposive sampling from numeracy teachers I previously worked with in my capacity as a numeracy teacher educator. Although the relationship between me (researcher) and the teachers introduced some bias, it also maximised the likelihood that the teachers were comfortable in allowing me access to their classrooms and their learners, and to discuss their approaches, thoughts and beliefs. The teachers were specifically chosen because I had previously seen them actively endeavour to link the mathematics taught inside the classroom with the potential uses that learners may have to apply their numeracy outside the classroom.

To maintain confidentiality and anonymity, teachers are referred to as Teacher 1 and Teacher 2, their learner groups, Group 1 and Group 2 respectively; learners who were interviewed were given pseudonyms and learning locations have been withheld. Having obtained ethics approval from Anglia Ruskin University, permission from the participants' learning organisations, and informed consent from the two teachers and their learners, data were collected from discrete numeracy classes in two different Adult and Community Learning settings towards the end of the academic year, once relationships between teachers and learners had been established, and once learners had some experience of learning numeracy in an adult class. Learner Group 1 were on a Family Learning course, designed to help them relate the numeracy they had previously learned on an Adult Numeracy qualification-based course to their children's key stages, and to explore ways they could support their children's learning. Learner Group 2 was attending a general numeracy class with an Adult Numeracy qualification outcome as its primary goal.

Teacher interviews, learner group interviews, and observation of classes were used to collect data. Semi-structured interviews were held with each teacher to investigate their aims and methods. The interview sessions were audio-recorded and the recording was fully transcribed for the purpose of analysis. For each teacher, two two-hour observations of their teaching were carried out, to observe the methods teachers use to help learners make links between their numeracy learning and the use of numeracy outside the classroom. The purpose of carrying out the observations was to enable verification of what teachers said they did at the interview stage (Robson, 2011), and to capture approaches that may not have been voiced during interview. Field notes were made during the observation to record non-audio information and audio-recordings were made using a digital recorder,

to minimise intrusion. Relevant parts of the audio recordings were transcribed for the purpose of analysis, and integrated with the field notes.

A short focus group interview was undertaken with willing learner participants (the invitation was open to all) from each learner group, to ascertain their perspectives. These were also audio-recorded and transcribed. As part of an established learning group, learners were used to talking as a group, and Brown (1999, p. 115, as cited in Robson, 2011, pp. 295-6) suggests that a homogeneous group is beneficial in giving a sense of safety and facilitating communication, although it may lead to unquestioning similarity of views. Care was therefore taken to ask open questions, and learners were asked to be honest with me, to minimise the effect of any bias I might bring to the interview (Sim, 1998, p. 347, as cited in Robson, 2011, p. 296).

The family learning class (Group 1) was comprised of five female learners, between 20-45 years old. All were mothers of young children. **Jackie** and **Emma** (pseudonyms) were involved in the interview. The general Skills for Life Numeracy class (Group 2) was comprised of thirteen learners (including two males) between 19-75 years old, some of whom had joined the 30-week class during the year. The four learners involved in this group's interview were given the pseudonyms **Barbara** (mother), **Carol** (mother), **John** (unemployed) and **Maureen** (retired).

A thematic coding approach was used as the basis for analysing the transcribed interviews and observation notes, to identify themes arising (Robson, 2011). The themes arising under coding 'teachers' aims' and 'learners' motivations' are listed in Table 1. Examples of the themes arising under 'Learner Gains' are identified in the Findings section below, and they have been used as evidence to inform Table 1, i.e. whether or not teachers' and learners' aims have been met.

To identify the types of activities used in practice, some pre-determined codes were identified at the outset, informed by a review of the literature and prior experience, but these were amended and other codes arose during data collection and analysis, on the basis of the research findings. Corbin and Strauss (2008, p. 66) liken the process of coding data to "'mining' the data, digging beneath the surface to discover the hidden treasures contained within data." This approach was essential in minimising researcher bias and in seeking to represent the data as truly as possible. Finally, the types of teaching and learning activities were categorized according to how 'abstract' they were, i.e., devoid of any non-mathematical context, and, at the other extreme, how 'situated' they were, i.e., immersed in a real-life context. Categories that sat between these two extremes were also identified during coding and analysis, e.g., I used the term 'Quasi' to describe the kinds of mathematical word problems which are included in mathematics and numeracy textbooks, worksheets and test/exam questions, but which commonly bear little resemblance to real life (Dowling, 1998; Oughton, 2009). The number of occurrences of different types of activities that were either observed or outlined by teachers during their interviews was used to establish the extent to which the two teachers used similar or different types of activities, and comparisons between the two teacher/groups were made. The order in which different types of activities were sequenced during classes was also analysed and compared.

Findings

Before exploring the kinds of activities and methods that teachers used, a summary of the findings that emerged from the study relating to teachers' aims, learners' motivations and the identified learning gains, is given below.

Teachers' aims and learners' motivations and learning gains

Table 1 summarises the data obtained from interviews with teachers and learners regarding their aims (in the left-hand column) and judges the evidence of their achievement (in the right hand column), by comparing the data analysed under the theme 'Learning Gains'.

Table 1.

Table Comparing Teachers' and Learners' Aims with Actual Outcomes

Teachers' and learners' aims	Learners' Gains – Is there evidence this has been achieved?
Teachers' aims	
Change learners' identity from "I can't" to "I can" learners.	Yes, including gains in confidence.
Enable learners to pass their numeracy test	Unknown (not in scope).
Enable learners to apply their new maths to real life contexts	Yes – some already in place, some planned.
Enable learners to better support their children's maths learning	Yes, and children improving their maths as a result.
To promote autonomy in learning / more independent learners	Unknown (not in scope).
Learners' aims	
To understand the maths their children are learning at school to be able to more effectively support their children's maths learning. (See 4. above)	(See 4. above)
To gain a qualification (See 2. above)	(See 2. above)
To face up to maths demons – to do the maths that couldn't previously do. (Links to 1. above)	Yes – evidence of better conceptual understanding.

As expected, learners' stated aims (See Table 1) for joining a numeracy class were similar to those in other adult numeracy research studies (for example, Coben et al., 2007; Swain et al., 2005; Swain & Swan, 2007;). Similarly, 'Helping with everyday things outside the classroom' was one of the least popular reasons identified in others' research, and it was not stated as an aim for the case study learners.

During the group interviews, learners were asked whether they used maths before they attended their class, and if so, what kinds of things they used it for. Not unsurprisingly, their responses varied, e.g. Jackie identified she did basic budgeting but explains:

> I would have veered away from things like percentages and that. If I wanted to know what something was off, I would ask my husband, rather than actually try and work it out myself.

Other responses included:

> Carol: You use it without realising you're using it, don't you? Checking change in the shops, checking receipts.

> John: I used to ask people. I never used it…I'd never try because I'd know I'd either make myself look really silly or get it completely wrong, so I just wouldn't bother.

> Maureen: I was okay with money and things like that…But anything like working out how much carpet I needed, or wallpaper and stuff like that, I would always give it, say, to my husband: 'How much do you think we need?'

After they had each responded to this question they were asked whether, since they joined the class, they had used any of the maths they had learnt to do something outside the classroom, and if so to give some examples. Responses included:

> Jackie: I don't shy away from working out percentages now… I try and work them out now. I do still get my other half to check them….And when the children come with problems with their maths at school I feel more confident to tackle that, so they're getting a bit more confident because they can do it as well.

> Carol: Sorting out finances and things. I would have left it all to my husband, but, that's my job now, working out things like how much my car costs to run… Since doing all this [maths class] I think how can I use it?

> John: I work out the bills now. She [his wife] says to me sometimes: 'Do you want me to work it out?' and I say 'no, I'll work it out.'

> Maureen: I'd measure the room and actually think about working out those calculations for myself [how many wallpaper rolls] and then maybe say to my husband: I think so-and-so, what do you think? Whereas before I'd never have done it myself.

Emma and Jackie also gave examples of how they plan to use their new maths knowledge and skills to help them with future DIY[2] projects, such as, designing a patio or floor (Emma) and interior design (Jackie), things they identified they would not have attempted previously. In summary, a range of themes emerged from the resulting data, including: metamorphosis from an "I can't" (do maths) to an "I can" person, gains in confidence, being confident with their new knowledge to help their children with their maths learning, their children improving their maths, better conceptual understanding and using (additional) maths in their everyday and/or working lives.

This research suggests that not only do the learners leave their course with benefits they had anticipated, but additional benefits are evident; in particular they are able to apply their new maths skills to their real life contexts. Therefore in answer to the research question: 'To what extent are teachers successful in enabling learners to apply the maths they learn?' it is evident that both teachers have been successful in enabling their learners to apply their maths skills, despite the complexities associated with the processes of learning and transfer.

Methods employed by teachers to help learners relate in-class learning to real-life uses

So how did the teachers achieve this? The methods that teachers used in their classes were analysed using a model I named 'The Context Continuum' (See Table 2), which I developed during the data analysis stage, from an adaptation of Kelly's (2009, 2011) research model (Presented earlier – see Figure 1).

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[2] Do-It-Yourself.

Table 2.

Table Explaining the Context Continuum Categories

Category Name (& Code)	Description of Category	Examples (from research data)
Abstract (**R_0**)	Instances where the focus is on the numbers themselves, or on the underlying mathematical patterns, relationships and concepts, where the learning and teaching is devoid of any context other than mathematics.	*Learners designing their own fraction walls.* *Submerging 3-D shapes in water to calculate volume and discover 1ml = $1cm^3$.*
Quasi (**R_1**)	The kinds of mathematical word problems regularly included in maths and numeracy text-books, worksheets and summative assessment questions which are written as 'contextualised' questions but which commonly bear little resemblance to real life contexts.	*Learner trying to make sense of frustum volume formula to answer worksheet problem.* *When using a 'saving water' worksheet, learners identify that the example bath is rather large.*
Validating (**R_2**)	The occasions where teachers encourage and validate learners' informal calculation methods, and where learners show that they understand that their informal methods are legitimate.	*Teacher using whiteboards when doing mental maths, to encourage learners to use and identify informal methods they use.* *Learner 1 used traditional method of multiplication and got the wrong answer. Teacher asked how else they could calculate it. Learner 2 used informal chunking method (and got right answer).*
Making Links (**R_3**)	Instances where learners and/or teachers discuss and identify links between mathematics or numeracy and its real life uses. This includes discussion of mathematical terminology, identifying specific mathematical meanings compared with general meanings.	*Teacher sharing news items with learners, e.g. doctors and new ratios of doctors to patients.* *Learners thinking in pounds (£) to make sense of decimal numbers.*
Re-creation (**R_4**)	Teachers' attempts to re-create real life contexts within the educational setting. Often involves using real artefacts. Possibly context may not be directly relevant to learners' lives.	*Learners working out the volume of compost needed to fill some real plant pots.* *Using external surroundings to find examples of tessellation and angles.*
Situated (Code **R_5**)	Closest to Real Life. Where learners and teachers provide/use examples of their actual uses of mathematics within a real life context.	*A learner being able to check her payslip (presented to her in hours and decimals, rather than in hours and minutes).* *Learners reducing recipe when making playdoh.*

Education (Mathematical) context

Real life context

R_0 R_1 R_2 R_3 R_4 R_5

Abstract

Concentration on the numbers / the mathematical concepts themselves, devoid of any context other than mathematics.

Validating

Teachers validating learners' informal methods of calculation.

Making Links

Learners and teachers discussing/ making links between mathematics and real life uses, inc. terminology.

Situated

Learners' and teachers' examples of actually using mathematics within a real life context.

Quasi

Use of mathematical word problems or summative assessment example questions which are 'contextualised' but unrealistic.

Re-creation

Teachers' attempts to re-create real life contexts in an education setting.

Figure 3. The Context Continuum.

Kelly (2009, 2011) focussed on workplace learning and compared education contexts with workplace contexts along one continuum (horizontal), and approaches to numeracy teaching along the other (vertical axis), from discrete concepts to problem solving. Because the methods teachers use to link classroom teaching to real-life uses form the basis of this case study, I focussed solely on the horizontal (contexts) axis, developing it to fit numeracy teaching that used 'real life' in general, rather than specifically a workplace context. Hence 'Real Life' context replaced 'Work Life' context. Based on the themes arising from the data, I divided the continuum into six subsections, hence 'The Context Continuum' (Table 2). Because my study was based in an educational context it also seemed more logical to start with the Education (mathematical) Context on the left. Codes were used to classify and sort the data, with 'R' representing 'Real', R_5 being as Real Life as possible and R_0 being devoid of any Real Life links. To illustrate, alongside their descriptions, examples (from the data) for each category in The Context Continuum is given in Figure 3.

For analysis, the specific examples ('occurrences') of numeracy teaching/learning methods identified throughout the data, i.e. in teacher and learner group interviews and class observations, were listed in a table and then the occurrences were sorted according to the six categories outlined in The Context Continuum. Although essentially a qualitative study, some quantitative analysis provided a useful means of identifying similarities and differences in teachers' methods.

The pie charts in Figure 4 below show that the number of occurrences of all but two of the different categories were similar across both teacher/groups, with 'Making Links' and 'Abstract' being the most prevalent for each teacher/group. This identifies that the instances of teachers and/or learners discussing and making links between real-life contexts and the mathematics they are exploring in the classroom form the highest occurrences of methods captured. Also focus on the abstract, including the development of conceptual mathematical ideas, forms a high proportion of the methods identified.
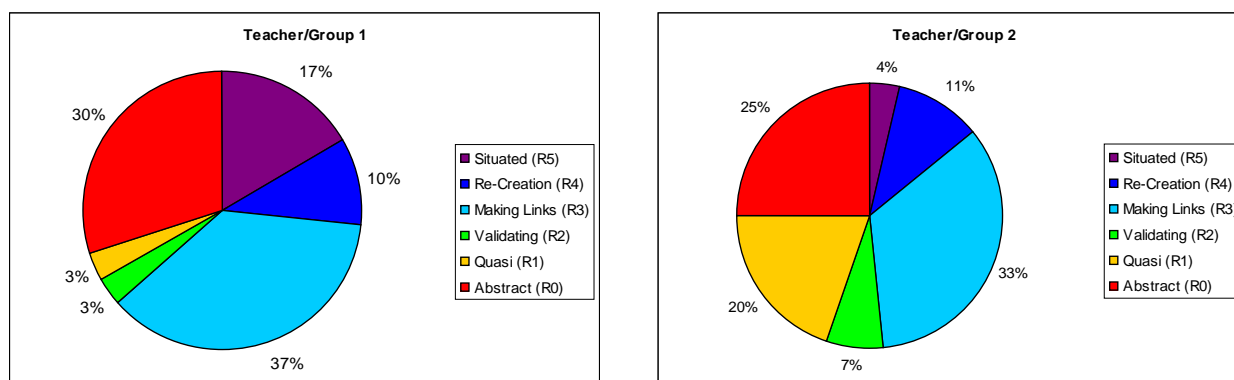
Figure 4. Pie charts showing analysis of methods by context, by teacher/group.

However, the next most-occurring category differed. A higher number of 'Quasi' methods (in orange) was listed in teacher/group 2 (20%) compared with teacher/group 1 (3%). A possible explanation is that in teacher/group 2 learners were working towards a national test as part of their intended course outcome, therefore focus on 'Quasi' methods such as practise test questions and typical word problems could be seen as conducive to supporting success in learners' test results. This contrasts with teacher/group 1 who were undertaking a short course for which there was no qualification outcome, and therefore 'Quasi' methods were likely to be less relevant in this situation.

In contrast, 'Situated' methods (in purple) made up a higher proportion of methods in teacher/group 1 (17%) compared with teacher/group 2 (4%). A possible explanation for this is that as a smaller, family learning group, all with young children, Group 1 were likely to have more common interests than the general Skills for Life group. These factors are likely to have made it easier to situate the learning in relevant real life contexts. Of course, it is also possible that these differences might be linked to teachers' naturally different styles.

Finally, aspects of the data were preserved in linked sequences within the analysis tables (matching sequences of activities either observed or discussed in interviews), so that the sequences of different classroom activities could be analysed for any patterns emerging. These sequences are presented visually in Figures 5a and 5b below.
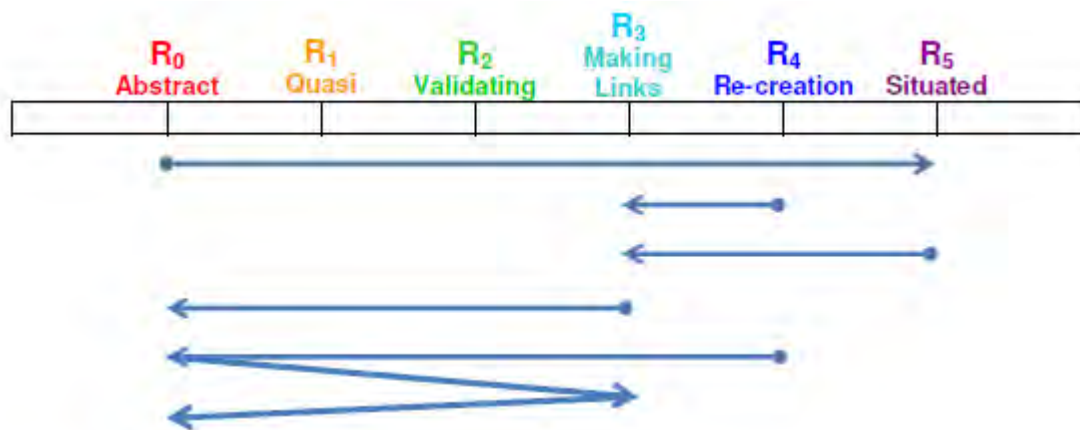
Figure 5a. Sequences of activities: teacher/group 1.
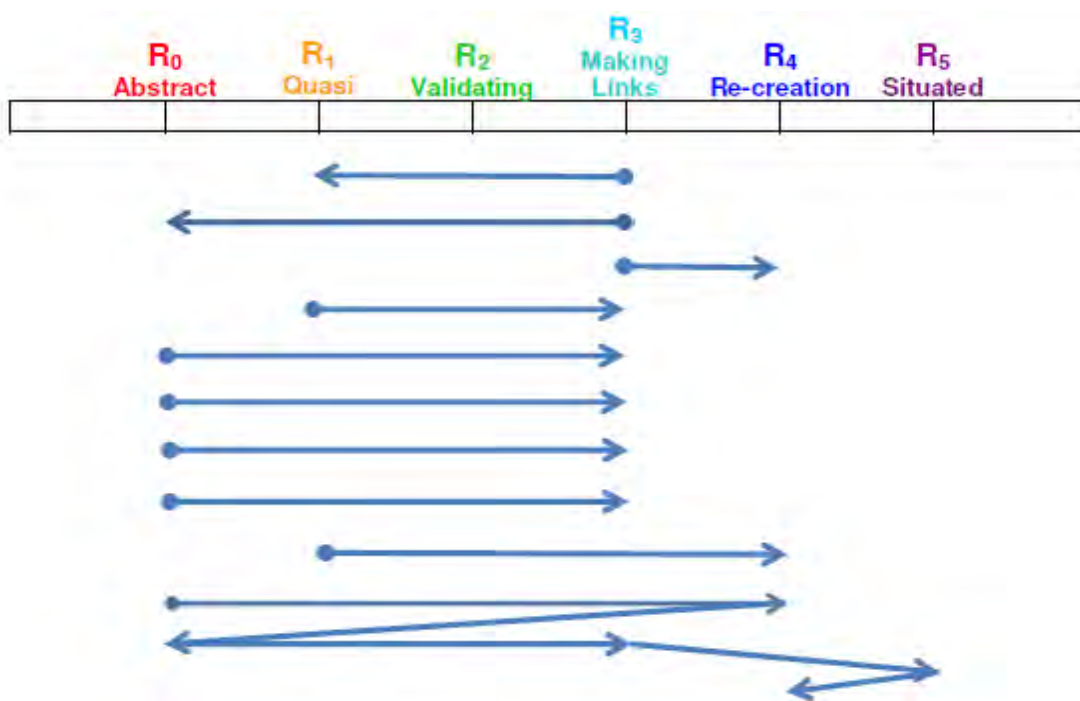
Figure 5b. Sequences of activities: teacher/group 2.

Analysis of the sequences of classroom activities shows that in general teacher/group 1 tended to move from the right hand side of The Context Continuum (Real Life context) (Figure 5a) to the left hand side of The Context Continuum (Education (mathematical) context). In contrast, teacher/group 2 generally tends to move from left to right along The Context Continuum, i.e. from an Education (mathematical) context towards a Real Life context. The differences in direction are consistent with the differences in methods identified. For teacher/group 2, focus on the quasi-mathematical word problems, which are prevalent in adult numeracy test questions, is a key focus, therefore it perhaps makes sense to start with the more abstract and move towards more real life contexts. In contrast, for teacher/group 1, opportunities to focus on the class's situated, real life uses of mathematics provided

a more important focus, hence starting on the right-hand side of the continuum and moving left perhaps makes more sense.

Data from the teacher interviews suggest there may be other contributing factors. For example, Teacher 1 had less positive experiences of maths learning at school, so from first-hand experience she is aware of how a lack of understanding and feelings of failure can have a long and lasting effect on people. A focus on the individual, or the personal as the starting point for her teaching would therefore make sense. In fact Teacher 1 explained that she "always start[s] with what they know" and that she avoids 'contrived' contexts that are likely to be meaningless to some learners. In contrast, generally Teacher 2 seemed able to make her own sense of the maths she was taught, whether or not it was related to a context. Consequently she did not appear to have any real barriers to learning mathematics herself. Teacher 2 explains how in dealing with a group of learners with diverse interests and experiences, she tries to give multiple examples, to do things that people are familiar with in some way, to hopefully enable them to make their own links. Therefore the data suggest that teachers' personal experiences, as well as learners' contexts and course aims, may influence their approaches to teaching.

Discussion, implications and conclusions

The findings identified that both teachers in this study enabled learners to apply their new numeracy knowledge and skills to real life contexts, in addition to a range of other aims. These findings are relevant to other numeracy teachers including those in different contexts (e.g. vocational teachers) as well as those in different countries, where the drive for qualification outcomes sits alongside educators' aims of making the learning useful beyond the qualification outcome.

In particular, this paper set out to answer the following questions:

- To what extent are teachers successful in enabling learners to apply the mathematics they learn?

- What methods do teachers employ in order to help bridge the gap between abstract mathematics and useable numeracy?

Whilst unsurprisingly (Coben, 2000; Swain et al., 2005), applying numeracy in real-life contexts was not an aspiration for learners on joining their course, nonetheless teachers' methods increased learners' knowledge, understanding and confidence and also their awareness of, and their ability and motivation to use their mathematics knowledge to support activities and problem solving in their own lives.

The way both teachers achieved this was primarily by focussing on developing the underpinning conceptual understanding of learners as well as focussing on linking these abstract procedures and concepts to real life applications relevant to learners. The differences in approaches, where Teacher 1 placed emphasis on situating the mathematics in learners' real life contexts and Teacher 2 placed emphasis on using quasi (contextualised but perhaps unrealistic) examples, could be explained by different course outcomes, learners' contexts, as well as teachers' own experiences of learning mathematics in shaping their beliefs and approaches to numeracy teaching. Both teachers used all six approaches identified on the Context Continuum. Further research would be needed to establish whether the course outcomes, learners'contexts, or teachers' natural approaches were more influential in guiding their course planning, and to what extent these approaches could be adapted or developed.

The different sequences of activities suggest that it does not necessarily matter which direction the order of learning activities occur, i.e. starting with real life situations and moving to the abstract mathematics within them or the other way around, but that either way, learners can gain the confidence and skills to use the maths they have learned *in class* for uses *outside* the classroom. Therefore it seems that, along with developing conceptual understanding, the important factor in the teachers' success is the movement back and/or forth between the difference discourses of mathematics and numeracy (as discussed by FitzSimons, 2006), embedded in discussions with learners which make explicit the links between formal and informal uses of mathematics.

Both classes were discrete numeracy classes in that the learning focussed on numeracy rather than being embedded in a vocational or workplace context. As a result learners had a wide range of backgrounds and experiences, and this sometimes made it difficult for teachers to identify contexts which were truly relevant to all learners. This perhaps explained why Teacher 1 was able to use more situated learning activities, as her learners were all mothers of young children, so they at least had this in common, and learning could be situated in the context of mother-educators supporting the development of their children's numeracy learning. Teacher 2's solution to the diversity was to offer a range of different contexts on a regular basis. Both strategies seemed to work, which suggests that a vocational or workplace context is not essential and that discrete numeracy classes can also support development of numerate skills and attitudes, as long as the teachers have the skills, commitment and drive to make their classes useful to learners beyond any qualification outcome.

The Context Continuum model was useful in classifying the different types of activities used in these discrete numeracy classes so that teachers' approaches and their similarities and differences could be explored and communicated. In using the model again I would consider additionally capturing and analysing the length of time spent on different types of activities. This was not appropriate in the pilot study as, in addition to observation data, the analysis considered interview data, i.e. it included discussions about un-recorded sessions and activities carried out on previous occasions with the same groups. I hope that the model, along with the findings, will be of use to other numeracy teachers or numeracy teacher-educators, and researchers, to support analysis and development of adult numeracy teaching practice. The continuum could also be adapted to be of use to practitioners and researchers of other subjects (e.g. literacy).

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Learning Mathematics Inside and Outside the Classroom

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Abstract

There is growing evidence (Lynch & Last, no date) that mathematics achievement can be a good indicator for future academic performance while studying at third level. This article reports on a pilot study carried out in an Irish university on Access to Science and Engineering students. The pilot examined the effects of an online adaptive learning tool for mathematics on students' confidence with the basics of mathematics. A pre- and post-diagnostic test was used before and after the students were given 40 days access to the personalized learning tool RealizeIt[1]. Following their semester 1 mathematics exam, semi-structured interviews were conducted to ascertain the students' experience with the technology.

Keywords: blended learning, mathematical confidence, adults learning mathematics

Introduction

The University College Dublin (UCD), Adult Education office has been offering the *Access to Science & Engineering* programme since 2001. Until 2013, when a new mathematics curriculum and a scientific enquiry module where introduced, the programme remained largely unchanged. From a pool of up to 100 prospective candidates seeking a return to education, approximately 30 are accepted in the course. The criteria for acceptance include:

1. No formal 3rd level education (exceptions for interrupted learning – life-changing circumstance etc.);

2. Evidence of some research and knowledge of the course;

3. Display self-motivation and a progression plan following the courses' completion;

4. Interest in reading popular science books; and

5. English competence etc.

Evidence of learning in a formal setting in the previous 3-5 years is a strong indicator of success in the programme. Prospective students are invited to attend pre-entry 'Hot topics in mathematics' workshops, and are also asked to submit a piece of writing, based on a lecture in some aspect of science, which they attend in UCD.

All candidates are obliged to take a diagnostic test in mathematics (without the use of a calculator), which assesses basic knowledge in arithmetic, algebra and statistics. Students are also invited to an interview, where they also learn about the course requirements and acceptance criteria. Acceptance in

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[1] See http://realizeitlearning.com.

the course is based on the student displaying evidence of how they meet the above criteria, and whilst the mathematics score is not an absolute disqualifier, the student may be cautioned against taking the course and advised to take additional tuition in advance of taking a place in the programme. After gaining entry in the programme, students study modules in Biology, Chemistry, Study Skills, Enquiry Skills and Mathematics 1 and 2, with both the mathematics modules being mandatory. In total, students are offered 72 hours of mathematics tuition.

Following consultation with the College of Science, the School of Mathematical Science and the School of Electrical, Electronic and Communications Engineering at UCD, the Adult Education centre decided to re-design the existing Access to Science and Engineering course with a particular emphasis on mathematics. In the previous 12 years only those students wishing to pursue entry on to an Engineering degree were obliged to do both of the mathematics modules (one during each semester) while those wishing to pursue a Science degree could choose to do maths in semester 1 only. From 2013/14 this condition was changed to the current one where *all* Access students must do both mathematics modules.

Syllabus changes to the mathematics content meant that two chapters on statistics and probability were introduced for the first time with none of the previous content to be removed. Changes to the exam were also made to exclude the option of exam question choice thus precluding a student from omitting to study certain sections of the course. The questions on the exam were also to be of a more applied nature and less theoretical. These changes are in line with developments made to the end of secondary school state examinations in mathematics in Ireland under a new initiative called Project Maths[2].. For more on the impact of Project Maths' at second level on the learning of a data analysis module at third level see the author's paper at http://www.projectmaths.ie .

Pre and Post-tests

Prior to admission on to the course all students were required to take a diagnostic test on the basic of mathematics including arithmetic, basic algebra and statistics. This test consisted of 15 multiple-choice questions and was done using pen and paper. Figure 1 below shows the results of the pre and post-test for each student. We were not interested in the actual scores on the exam but rather in the trajectory of improvement (if there was any). The post-test was conducted 6 weeks after the pre-test with the questions remaining the same but with some of the post-test questions including algebraic symbols instead of the numbers used in the pre-test.

During the gap between the pre and post-tests the students were given access to the online interactive maths tool RealizeIt for 40 days. The software was not imbedded into the course and it carried no course credit. Students were told that this was an optional extra resource, which they may avail of, or not. The students were given a brief introduction to the software system in a dedicated Access slot at the UCD Maths Support Centre (MSC). The results of the pre and post-test are given in the following table labelled figure 1.

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[2] See: http://www.projectmaths.ie.

Student	Pre-test (out of 15)	Post-test (out of 15)
1	3	13
2	3	9
3	3	6
4	4	11
5	4	10
6	5	13
7	6	15
8	6	14
9	6	11
10	6	10
11	7	15
12	7	14
13	7	14
14	7	13
15	8	14
16	9	15
17	9	14
18	10	15
19	10	15
20	10	13
21	10	11
22	11	15
23	11	14
24	11	13
25	13	14
26	13	14

Figure 1. Pre and post-test results

Pre-test

The Diagnostic Test played a role with regard to the software also, as the use of the software was restricted so that the students could only have access to 4 nodes, namely:

1. Indices

2. Linear Equations

3. Functions &

4. Statistical Models

One of the key features of the software is *Determine Knowledge.* This is where the student is asked questions about how well they think they know the material from a particular node. The system then delivers a series of questions (roughly ten) based on their answer. This then sets them on their own personalised path of learning where the aim is to cover an objective by mastering all the associated nodes. If the system senses that a student is struggling it will prompt them to try a prerequisite node to the one they are currently on.

Thematic Analysis

Following the students 40-day involvement with the online tool all those who used the tool (*used* here meant that they logged in for a minimum of 5 hours throughout the 40 day period in question) were asked to attend a semi-structured interview with myself to discuss their experience with the software. Seven students participated in the interviews. Upon thematic analysis of the interviews 4 emergent themes were identified. These were:

1. Usage of the system, including how long, how often and user experience

2. Usefulness

3. Improved knowledge/confidence in the basics of mathematics

4. A desire to use it for other modules

While only seven students participated in the focus group interviews the following findings were discovered with reference to the four themes listed above. Under *usage of the system*, students said often and short was the key, meaning that they used it every day or every other day and for periods of between 19 and 42 minutes at a time. In terms of user experience each interview participant said the system was very easy to navigate. The following are transcribed student comments.

- I used the software for about half an hour every day, I thought little and often would help it to sink in better.

- I used the system 34 times in total – usually for a period of 30 mins to an hour each session.

- I'd usually try to use if for around a half hour at any one time.

- I would use this system maybe once or twice a week. I would use it maybe for 2-3 hours when I did.

All seven students found the system to be very useful with special reference being made to the suitability of the system to *filling in gaps* of knowledge that may have been lost through time. Students said:

- I found that the explanation and the practice questions made the subject stick much better in my mind. I think I'm a visual learner and its beneficial to have a system that allows you to take part in the demonstration to really break down the subject material to a level where I can understand it.

- It broke topics into easily managed chunks and you could only progress after reaching required level in basic topics. I liked that it gave you a colour coded chart of how well you had performed on each topic.

- I found the system really useful when starting a new topic in maths or when revising a topic to highlight and fill in the gaps in what I knew.

All interview participants found that the system helped them improve their *confidence* with the basics of mathematics. They said:

- Yes, it greatly improved my ability and confidence in quite a short space of time, my husband has always made fun of my maths abilities and now I can do things he can't! I definite put it down to this programme. I don't think I could have achieved such a good mark in my maths assignments without it; to have it at my fingertips was great. You can only do the practice sheets so many times before you remember the answers, but for the most part the programme had a large catalogue of questions.

- Yes, definitely, when I used it for topics like that I knew I was generally weak at, I noticed that after a while, I was able to answer the questions faster and more easily.

- Yes it increased my knowledge and understanding of indices and helped me to get my head around the basics.

With reference to the final theme of *a desire to use with other modules*, some student comments include:

- I would very much like to have a tool like this with other subjects. This type of tool could be really useful for stoichiometry in chemistry. I found the visual aid useful to have when I got stuck working through a problem and I didn't have to search through a load of video demonstrations on the internet.

- Yes most definitely, I certainly wouldn't solely rely on it for my maths learning, but it is a great tool to go to when having issues and to help with understanding any areas of trouble. It could only be a good thing to have this resource on other maths related subjects.

- Yes, I think it would be really useful to have this type of resource for use with other subjects. There are so many different web sites and online resources but it is very helpful to be supplied with a resource by the maths department, it takes a lot of the guesswork out of whether it's a good resource or not.

Conclusion

While we stress that this was a pilot conducted using 26 students and with just 7 students completing the interviews we are very heartened by the level with which students confidence in the basics of mathematics increased in a relatively short space of time (the students used the system for 40 days and there were just 66 days between the pre-test and the interviews). Adult students take a strong part in their own learning and are intrinsically motivated[3] so having such an effective online tool can make the transition from working life to university that little more manageable. In the future we hope to increase the sample size for the study to include modules from across the university taken by mature (mature at our institution means of age 23 or above on the first of January in their year of registration to the university) and adult learners that have a mathematics related module in their degree stream.

In this time of promised educational revolution due to new technologies, MOOCs, blended learning, flipped classrooms etc. it is of value to state that a mix of face-to-face and out of classroom activities via an online tool can be an effective means to increasing students confidence with their mathematics. Recent research (Howard, 2014) has shown that students find that meeting in a Maths Support Centre environment can have a positive impact on students learning and identity of mathematics.

A recent study of Means et al (2013) found that:

> *On average, students in online learning conditions performed modestly better than those receiving face-to-face instruction. The advantage over face-to-face classes was significant in those studies contrasting blended learning with traditional face-to-face instruction but not in those studies contrasting purely online with face-to-face conditions.*

The same paper concludes that:

> *Studies using blended learning also tended to involve additional learning time, instructional resources, and course elements that encourage interactions among learners. This confounding leaves open the possibility that one or all of these other practice variables contributed to the particularly positive outcomes for blended learning. Further research and development on different blended learning models is warranted. Experimental research testing design principles for blending online and face-to-face instruction for different kinds of learners is needed.*

At this conference we hope we have given an example of a *blended learning* pilot whereby a little creativity in the use of a Maths Support Centre's time and resources can result in an effective and efficient result for both students and instructors alike.

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http://www.projectmaths.ie

http://www.ucd.ie/adulted/tutors/tutorresourcematerials/adultlearningstyles/

Exploring Student Perceptions of 'Real Life' Contexts in Mathematics Teaching

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Abstract

The use of 'realistic' contexts for mathematical problems seems to offer a means of making connections between the classroom and the world outside but these are often interpreted in different ways by students and may lead to some unexpected responses or outcomes. Instead of demonstrating the relevance of mathematics, questions may appear contrived or unauthentic to students, thereby increasing the distance between classroom mathematics and 'real life' rather than bridging the divide. This interactive workshop provided an opportunity to examine samples of mathematical tasks in different contexts and discuss issues regarding the authenticity and relevance of these activities for young adults. Research findings, from focus groups with vocational students in three Further Education colleges, were used to explore how students themselves might view the tasks and a three-dimensional framework for student perceptions of relevance, developed from the research findings, was used to facilitate discussion on how these sample tasks could become more effective.

Keywords: mathematics, contextualised, relevance, authenticity, vocational

Introduction

Workshop participants firstly discussed the following sample task and suggested possible answers to the question.

Shelley wants to make a chilli for her friends. The following basic ingredients would be enough for 4 people:

Minced beef	500g
Tomatoes	300g
Red kidney beans	300g
1 large onion	
1 red pepper	

Shelley has 0.8kg of mince. How many friends should she invite to dinner?

A number of answers were discussed, which were based on different interpretations and assumptions about the task. When considered as a practical problem within a 'real life' situation, questions about whether Shelley had sufficient supplies of all the other ingredients, or could easily obtain further supplies from a local supermarket, became concerns that in 'real life' could provide alternative answers to the question of 'How many friends should she invite to dinner?' Discussion about the actual question and possible interpretations led to a consideration of how realistic the whole task was and whether determining the number of friends to invite to dinner on the basis of the ingredients available was a typical approach to arranging a small dinner party. Adapting the recipe and going

shopping to obtain more of the ingredients were discussed as options in 'real life' but these were dependent on further information about the other ingredients available or the distance to the nearest supermarket. Without this information there were constrained choices for interpretation of the task.

Viewing the task as a mathematical problem that was disconnected from the 'real life' situation was another alternative that may occur in a traditional mathematical classroom. In this case the expected answer might simply be to invite five friends since there was sufficient mince to feed six people. These discussions introduced one of the fundamental difficulties in using context in mathematics to try to present a 'realistic' problem. Students may make different assumptions about the problem to be solved, as illustrated by the inconsistencies in children's responses and interpretations of 'realistic' tasks observed in various studies (Cooper, 2004; Cooper & Harries, 2002). Different approaches by students may or may not be aligned to the expectations or intentions of teachers in using contextualized questions and answers might be rewarded or discouraged accordingly.

Uses of context in mathematics

When contextualised mathematics tasks are used in different situations, inside and outside the classroom, then these acquire different meanings and serve different purposes. For example, the purpose of a classroom task may be to develop conceptual understanding whilst replication of a similar mathematical process in real life may be carried out to achieve a more personal goal. The situation in which the task is being used therefore affects the interpretation and, in a classroom, the role that a student adopts influences the way they approach the problem.

Wiliam (1997) explains how context may have varied uses but may, in some cases, be seen as a 'MacGuffin' and serve little purpose except to motivate students. According to Wiliam (1997), contexts used for mathematics teaching may be classified into three kinds as follows:

- Contexts that bear little relation to the mathematics being taught ('mathematics looking for somewhere to happen')

- Contexts having structure with components that can be mapped onto the mathematical structures being taught ('realistic mathematics')

- Contexts in which the main aim is to solve a problem in which no particular mathematics needs be used ('real problems')

Several sample tasks were then discussed in the workshop. These were taken from lessons in which vocational students were learning mathematics using contexts or scenarios that the teachers considered as relating in some way to their particular vocational specialisms or personal interests. The sample tasks included:

- '**Planning** a kitchen' for **construction** students. This involved planning and positioning various items of equipment in a kitchen, within given constraints and drawing a scale diagram of the floor plan.

- '**Mixing colours**' for hairdressing students. This required students to calculate the actual amounts of different hair dyes to make the required colour from given ratios.

- '**Alcohol awareness**' for students on a public services course. This used some simple summary statistics on alcohol consumption for different age groups in the UK as a source of data for questions on rounding large numbers.

These tasks were discussed in small groups and the following questions considered:

- Is this an authentic task?

- Is the context real? Is the context imaginable?

- What purpose does the context serve? Is the context necessary?

In these discussions the focus was on the workshop participants' own perceptions of the tasks as teachers or teacher-educators, whether these were authentic uses of mathematics and how they might be matched to Wiliam's (1997) three categories.

Secondly, three further questions were discussed for each task that moved the focus towards the student perspective, the purpose of using the task with a group of students and the expected outcomes:

- How do you think students would interpret and respond to this task?
- What conceptual understanding might be developed?
- What other learning might take place?

The main points emerging from these discussions were then compared with data regarding the opinions of vocational students obtained from a study of the experience of vocational students learning functional mathematics in three Further Education colleges in England. In this research, case studies of 17 different groups of students from the vocational areas of hairdressing, construction or public services and their mathematics teachers were developed to explore factors that affected the student experience of learning mathematics.

Student perceptions of contextualized tasks

Within the study outlined above, one of the research questions was concerned with the relevance of functional mathematics for these vocational students and, as part of the research, focus group meetings were held with students from each case study group to discuss three contextualised tasks from a bank of ten tasks. These tasks were unfamiliar to the group and they were not expected to attempt the tasks but discussed the following questions:

- What are your first impressions of this task?
- Is the task relevant to you in any way?
- Would you learn any useful maths skills or knowledge through doing this task?

For the three main tasks already considered in the workshop some of the comments from students were presented and compared:

Planning a kitchen

Although the task was primarily about drawing a scale diagram, some students identified the task as relevant to their intended occupation as a plumber or electrician simply because the tasks description included a sink unit (which the plumbers had to deal with) or a cooker (which some electrical students assumed would be electric and was therefore related to their vocational area). One student, however, rejected the task as irrelevant because it involved using centimetres (to measure the scale drawing) and justified his decision on the basis that only metres and millimetres are ever used in the construction industry. By selecting one or two words from the scenarios these students made judgments about the relevance of the context without considering the actual task involved.

Secondly, some students could imagine that they might carry out a similar task in the future to plan their own kitchen but believed the task was irrelevant because this was an event too distant from their current situation. In contrast one student stated that the task was entirely relevant since he worked for a builder at the weekend and was expected to assist with planning, sketching and sometimes making scale drawings as part of the job. It seemed that students who identified an immediate use for the processes involved were more likely to judge the task as relevant and their limited experience of workplace practices could significantly affect these judgments.

Mixing colours

This task also stimulated some mixed reactions from students in the research. Some quickly fixed their attention on the title and judged the task to be relevant because they had to mix colours in the salon.

Others looked at sentences within the scenario and made judgments based on a small section of information rather than considering the actual task. Some rejected the task on the basis of the terminology since the word 'rinse' was outdated and they would normally refer to 'quasi' instead. There were conflicting opinions about a reference in the scenario to tubes of dye. One group rejected this as irrelevant because they did not use tubes of dye in their training salon. Even though these were commonly used in salons this mismatch to their personal experience influenced their judgment about the relevance of the whole task. In a similar way to the construction students, inconsistencies between the details of the scenario and their personal experience, and the limitations of their current experience were important factors that influenced their perceptions of relevance.

Alcohol awareness

This task was generally thought to be relevant by students because it was connected to a topic of interest that they felt was an important part of their current lives. Most students made this judgment on the basis of the topic from which data was taken and overlooked the mathematics involved, which was only about rounding large numbers. This was a common approach across the students groups and, in most cases it seemed that the description of the context or scenario was used to make judgments about the relevance of the task rather than considering the actual mathematical processes or the tasks as a whole.

Similar themes were present in student responses to other samples of contextualised tasks. In addition students highlighted the importance of the apparent difficulty of a task in determining whether they would even attempt the problem, including the visual layout and whether the images used looked realistic.

Summary of student responses

As shown by these three selected samples, there were some themes within students' responses, but there were also some contradictions in their perceptions of the tasks, their relevance and purpose. Although some students stated that the context of a question made the mathematics more understandable, the contexts could also lead to confusion or mask the actual question. Using a vocational context could make the mathematical task seem more relevant but their limited experience could lead to misunderstanding about how realistic some scenarios might be. As a result, situations could be discarded as irrelevant by some students, whilst others understood the connections. Some students believed they would be more readily engaged by contextualized tasks but others had expectations about mathematics questions that led to tasks being discarded as irrelevant because they did not look like mathematics.

Although the assumptions and interpretations of these contextualised tasks by students varied, there were some themes within their responses that indicated how their perceptions of relevance were constructed. In general student perceptions of relevance in the research might be summarised as follows (Dalby, 2014).

Level of connection

Students viewed the tasks at different *levels,* which affected their judgments about relevance. These could be described as:

- Recognition of key words and their association with current life experience

    (Attempts to connect to the context through simple word associations)

- Recall of similar situations

    (Attempts to construct meaning from the situation described)

- Reality of the scenario

    (Attempts to interpret the scenario as an authentic part of real life)

Immediacy of usefulness

Student judgments were affected by their perceptions of whether the task had an immediate usefulness or would only be used in the more distant future. Tasks were judged as most relevant if there was a link to an immediate use in students' personal lives or their vocational course and less relevance if students had some expectations of encountering a similar scenario somewhere in their future lives. For some students however, mathematics was perceived as having no use at all once the course had ended.

Affiliation or interest

Different contexts evoked different levels of affiliation or interest depending on the strength of the connection to personal or vocational values. Students generally related strongly to situations that they could identify as part of their vocational specialism and to situations that they could identify as connected directly to their personal interests. Contexts that held little personal interest or were unconnected to the values associated with their current identity were seen as less relevant.

This led in the study to the following three-dimensional framework for examining students' perceptions of the relevance of contextualised tasks (Dalby, 2014):

- Level of connection (word, scenario or task level association)

- Immediacy of usefulness (current experience, expectation of future use or 'beyond the radar')

- Depth of interest or affiliation (vocational and personal interest or unconnected to personal values)

In the final part of the workshop, small groups discussed how they might adapt one or more of these tasks to increase the relevance for a chosen group of students.

Conclusion

These discussions highlighted how teacher and student perceptions of contextualised tasks and their relevance could vary and how different interpretations needed to be considered to avoid misunderstanding or unintended outcomes when used in the classroom. In a similar way to the studies conducted with children (Cooper, 2004; Cooper & Harries, 2002) some vocational students had expectations of mathematics tasks in a classroom situation that affected their interpretations and responses. Others, however, identified with the 'real life' scenario described, made assumptions associated with the situation in 'real life' and adopted a practical approach. Background and experience were key influences on their interpretations and perceptions of relevance. Student interpretations of contexts were highly influenced by their current situation and experience, suggesting that using context needs to be considered carefully in the light of the students involved and their life situation.

When students understood the relevance of the context to their lives then this did encourage engagement with the task and might therefore be considered as a source of motivation, as suggested by Wiliam (1997). It was, however, the context rather than the mathematics that seemed to engage the students in this study and there was little evidence that the structure of the task made any significant difference to their perceptions.

Acknowledgements

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Connecting Mathematics Teaching with Vocational Learning *

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Abstract

For many vocational students in England, mathematics is now a compulsory part of their programme, yet the inclusion of an academic subject within a vocational course presents challenges. In this paper, an analysis of a series of case studies of vocational student groups in Further Education colleges in England shows how contrasting practices in 'functional mathematics' and vocational classes reinforce perceptions that mathematics is an isolated and irrelevant subject. Some mathematics teachers made contextual connections by embedding mathematical problems in vocationally-related scenarios but distinctive socio-cultural features of vocational learning situations were often absent from mathematics classes. Addressing this disconnection requires a pedagogical approach and classroom culture that links mathematics learning with vocational values. The findings suggest that adopting mathematics classroom practices that reflect the surrounding vocational culture creates greater coherence for students and has positive effects on their engagement with mathematics learning.

Keywords: mathematics, functional mathematics, vocational education

Background to the study

The separation of vocational and academic pathways in post-16 education is a result of long-standing divisions that remain unresolved within the English education system (Young, 1998). Entry to the academic pathway is largely controlled by success in GCSE examinations at age 16 and those with low GCSE grade profiles often transfer to vocational pathways in separate institutions such as Further Education colleges. The divisions between the academic and vocational pathways are not only institutional but there are distinct differences in the curricula, qualification types and forms of knowledge associated with each strand. Students have constrained choices in post-16 education within a highly-stratified system (Pring et al., 2009) that tends to prioritise the academic over the vocational.

Within vocational education, both the mathematical skill levels of students and the qualifications undertaken have attracted criticism (Wolf, 2011). Historically, many low-attaining 16-year-olds have taken no further mathematics qualification by age 18 (DfE, 2014). Recent policy changes, however, now require these students to work towards re-sitting the GCSE mathematics examination until they achieve a grade C. When coupled with the recent extension of compulsory education to age 18 years in England, this means that many more students on vocational pathways now learn mathematics as a compulsory part of their study programme. This GCSE mathematics curriculum is traditional and academic in nature and so does not sit easily with their vocational learning.

The research reported herein was conducted prior to these national policy changes, with students who were taking a Functional Mathematics qualification rather than re-sitting the GCSE examination. Functional Mathematics focuses on problem solving and applications in 'real life' scenarios and, for most students in the study, the subject was compulsory due to college policies, although it was not a government requirement at the time. This paper examines the contrasts between mathematics teaching and vocational learning that emerged from a wider study of the students' learning experiences of mathematics and was primarily concerned with students aged 16-18 years. Before examining the research findings, some of the relevant historical academic-vocational tensions are discussed.

Divisions of knowledge, curriculum and pedagogy

The institutional divisions within the English education system can be traced back to the separate establishment of schools, work-related training and adult education. These educational traditions have continued without a coherent overarching policy for education as a whole (Maclure, 1991; Young, 1998). The academic and vocational education traditions that have grown from these roots have different purposes, curricula and qualifications but also reflect longstanding societal hierarchies (Hyland, 1999). The "links between the stratification of knowledge in the curriculum and patterns of social inequality and distribution of power in the wider society" (Young & Spours, 1998, p. 51) are evidenced in the privileging of academic over vocational pathways.

Low-achieving students often undertake vocational qualifications after the age of 16 although these qualifications are seldom considered in schools (Hodgson & Spours, 2008) where the focus remains on academic GCSEs, both prior to age 16 and after the parting of the academic-vocational ways. Vocational training in Further Education colleges has the twin goals of developing practical competencies and acquiring relevant technical knowledge in order to prepare individuals for employment. In contrast, academic qualifications in post-16 pathways prepare students for higher education and GCSE mathematics continues to act as a highly-valued 'gate-keeper'. Despite some attempts to bridge the divide by increasing the academic rigour of vocational qualifications or bringing vocational education into schools, these initiatives have historically had limited success (Hyland, 1999).

The teaching of academic and vocational subjects draws upon contrasting traditions (Lucas, 2004). For vocational education one of the major influences has been the close association with the apprenticeship model of learning, in which the teacher, as an occupational expert, demonstrates skills for students to replicate until they achieve competence in a 'community of practice' (Lave & Wenger, 1991; Wenger, 1999). Teachers may take a range of roles within vocational workshops and classrooms but practical activity is particularly important in a learning process that is essentially social and collective (Unwin, 2009); the emphasis is on developing competency within a community rather than acquiring knowledge (Hyland, 1999).

The academic strand within Further Education reflects a more classical, liberal approach to education in contrast to the practical usefulness valued by vocational areas. Robson (2006) argues that pedagogy needs to reflect the disciplinary context but this causes an uneasy relationship when a subject such as mathematics is taught as part of a vocational programme. Learning mathematics for vocational purposes focuses activity on a particular context and practical need but this utilitarian view (Ernest, 2004) is in tension with the broader appreciation of mathematics and abstract knowledge valued in academic pathways.

Vocational departments in Further Education colleges associate strongly with their particular occupational values (Robson, 1998). The tendency for students to adopt these values (Colley, James, Diment, & Tedder, 2003) suggests that students primarily focus on their vocational goals, resulting in perceptions that subjects with no clear vocational purpose are peripheral. Such values are key components of departmental culture but are also important influences in the teaching of mathematics (Bishop, 2001; FitzSimons, 1999). Against this background of historical traditions, our interest here is in the differences between students' experiences of mathematics and vocational learning, including the pedagogies and values enacted in these lessons.

Research questions and methods

The research questions of interest in this paper are:

- In what ways are students' experiences of learning in vocational sessions and Functional Mathematics classrooms related?

- How does this affect their learning of mathematics?

To answer these questions we compare teaching and learning approaches in mathematics and vocational sessions, using lesson observation data from a wider study of vocational students' experience of functional mathematics in Further Education.

The research involved a series of nested case studies within vocational areas in three Further Education colleges, from which cross-case and within-case comparisons could be made. Seventeen different student groups were involved from the vocational areas of Construction, Hair and Beauty, and Public Services and each student group formed a separate case study. The research was exploratory as well as explanatory and used multiple methods, both qualitative and quantitative, to provide triangulation between sources and methods. Drawing on ideas involved in grounded theory, an iterative process of analysis was used that involved the coding of qualitative data and constant comparison to identify emerging themes.

In addition to the lesson observations of the same student groups in Functional Mathematics and vocational sessions, data was obtained from student focus group discussions, interviews with Functional Mathematics teachers, interviews with vocational teachers, staff questionnaires and individual student card-sorting activities. In the card-sorting activities students either ranked statements, or placed statements on a Likert scale, to describe their experiences of school and college. In the following section we present some of the relevant results from these activities as background before examining the lesson observation data.

Research findings

When students ranked statements about their reasons for coming to college, the dominant reasons that emerged from the analysis were 'I was interested in the course' and 'I wanted to improve my education'. Focus group discussions provided further evidence that most students were interested in their vocational courses and valued the opportunity to choose the direction of their education, even though these choices were somewhat constrained by their GCSE profiles.

Secondly, students placed statements regarding their experiences of college on a Likert scale and discussed these in focus groups. Most students depicted college in positive terms (See Table 1 in Appendix I) referring to features such as being treated in a more adult manner, experiencing greater freedom, having more agency and taking more responsibility for their own learning. These results suggest that values relating to adulthood and employment were important to students and welcomed their presence in the college culture. In contrast, many students referred to their experiences of mathematics in school in negative terms (Table 2). Focus group discussions provided further evidence that most students approached college with a view that mathematics was a remote and irrelevant academic subject, one associated with previous failure and disaffection.

Within this context, where many students were positive about their general experience of college but showed an initial negative disposition towards mathematics, we compare their experiences in vocational and mathematics sessions. The differences will be set out using two short summaries of observed sessions. These exemplify the high contrast between vocational sessions and the traditional features of mathematics lessons that were evident in many of the seventeen case studies. After summarising the key features, the approaches used by some functional mathematics teachers to connect the two learning situations will be considered.

Observation A: Beauty Therapy students in the training salon

The students were giving facial treatments to clients. This involved individual skin consultations and one-to-one practical work. One student, Nina, was demonstrating the treatments on a "doll" (artificial head) to students who had missed the previous session. Nina explained the stages of the

facial and how each had to be completed properly but within a timescale of about 30 minutes since extra time would lose money for the business. Another student, Gemma, was acting as the salon manager: replenishing products, keeping records of the treatment times and generally making sure the salon was running smoothly. Quiet, relaxing music was playing as each student worked individually on their client. Several times during the session the students were reminded by the teacher to talk solely to their client and not chat to other students. All the students were wearing clean uniforms and seemed to have taken considerable care over their personal appearance. Students were expected to maintain their own uniforms, have their hair tied back and keep jewelry to a minimum. Apart from moderating the atmosphere, the vocational teacher watched and advised, acting as a guide and source of further information when necessary.

Observation B: Functional mathematics with Public Services students

The session took place away from the Public Services vocational area. Space was tight and although the students could all be seated at tables there was little room for the teacher, David, to move between them. This had an impact on the lesson since it was difficult for him to check work, give feedback and support individual students. After a formal teacher-centred introduction and some worked examples on the board, the main activity was to complete a series of worksheets about areas and perimeters. These were given out one at a time so that the completion of any worksheet was quickly followed by the provision of another. David tried to circulate to mark work and encourage students to participate but it was difficult to get students to engage with the work and frequent reminders were needed to keep them on task. He worked hard to keep distractions under control by reminding students to be quiet and get on with their work. These attempts to impose a working environment dominated the session and, despite being calm and persuasive, David's strategy seemed largely ineffective. Towards the end of the lesson students who had completed the work were allowed to go early whilst the others were retained and urged to continue until the official end of the lesson.

In the vocational session students were expected to adopt professional standards of behaviour and take significant responsibilities such as making decisions about treatments, supervising other students and providing customer care. In contrast, the Functional Mathematics lesson was a tightly structured, teacher-controlled session, closely resembling a typical school mathematics classroom. Students had little opportunity to make decisions about the learning process or take responsibility for their own progress as the whole process was largely controlled by the teacher.

Within the training salon there were clear rules regarding personal appearance and professional conduct but there was also considerable freedom. Students were expected to focus on their client during the session but walking around to collect equipment or products was part of the normal routine. Unprofessional chatter with their peers was prohibited but consulting with other students for support or advice was an accepted feature of working practice. In the functional mathematics classroom, space was constrained and students were expected to remain seated throughout the session. This created a very different environment and influenced the way in which learning took place.

David's approach to teaching was topic-based and the lesson involved an explanation of the mathematical content before demonstrating the processes through worked examples on the board. This was followed by student work on further examples that they were expected to complete quietly and independently. For David, mathematics should be learned in an organised, orderly classroom with clear rules enforced by the teacher. In contrast, the teacher's role in the salon was mainly to observe and advise. Students learned from one other as well as from their teacher in this collaborative and supportive environment.

There were further contrasts in the type of tasks used. In the vocational session practical skills and theory were integrated into tasks. For example, relevant theory about skin types needed to be recalled and used during consultations with clients. Tasks in the salon would generally take some time to complete and there was some flexibility about the time taken for each component as long as the overall treatment was completed within a reasonable timescale. Learning in David's classroom mainly involved short written tasks with the expectation that students would remain 'on task' and completion would be followed immediately by additional written work.

These two approaches to teaching and learning seemed to be based on contrasting values and assumptions regarding the role of the teacher, the environment and the processes that would be most

effective for learning. Relationships between the teachers and students in these two examples were very different, as were the social structures and classroom cultures in which roles and relationships were embedded. Comparisons with other observations of vocational sessions in salons, workshops and classrooms, showed that this session was very typical. This cross-case analysis led to the identification of four main areas in which there were common characteristics:

- *Responsibility, agency and freedom.* Students worked within loose frameworks of rules that related to health and safety requirements or professional standards but had freedom to make individual decisions. They were expected to take responsibility for their learning and were placed in positions of responsibility for clients or other students. There was freedom of movement around the vocational salons, workshops and classrooms.

- *Vocationally-related values and expectations.* Adult and work-related values, dispositions and behaviours were encouraged and evident in most sessions.

- *Student-focused learning through guided activity.* Learning processes centred on developing practical competencies through replication of skills demonstrated by respected vocational experts. Their role was to facilitate learning, with students acting as apprentices in a community where informal peer learning was often evident.

- *Integration of knowledge and skills into substantial tasks.* Practical skills were highly valued but knowledge from theory sessions was often intertwined into tasks. Tasks were usually substantial with multiple elements and time-scales stretching over days or weeks. Students worked at their own pace, making individual decisions about the order of the sub-elements and the methods to use.

These four areas contrasted with the formal, traditional approach to teaching mathematics in David's lesson where the following key features were identified:

- *Teacher authority and control.* The rules in the classroom reflected the values and priorities of the teacher-authority who expected students to comply. The teacher directed and controlled the learning process. Students had little agency in their work. They were expected to remain seated throughout the session, to work quietly, individually and follow directions.

- *Academic values and expectations.* The students were learning a subject as a series of disconnected topics, through a process of knowledge transfer rather than developing a set of skills.

- *Teacher-led activity.* The lesson was planned and closely directed by the teacher. Mathematical knowledge was delivered to students who did not aspire to be mathematicians and had little sense of how this learning might be useful.

- *A focus on written work.* There was a reliance on worksheets and written solutions to questions. The tasks were usually short and students were expected to remain 'on task' throughout the lesson.

Not all of the mathematics lessons were, however, like David's. We now consider those cases in which the Functional Mathematics teachers adapted to the vocational environment with lessons that were better connected to the students' vocational learning experience. The key features of these lessons are, again, presented using a short lesson observation as an exemplar, followed by a summary of the common characteristics of similar lessons from the cross-case analysis.

Observation C: Functional mathematics with Hairdressing students

The session took place in a separate building, some distance from the main Hairdressing area. As the students arrived the teacher, Richard, greeted them individually and engaged in relaxed conversations about what they had been doing both inside and outside college since the last class. His introduction to the lesson involved a class discussion about using units of time. Students readily talked about their difficulties, both asking and responding to questions until they were satisfied that they understood the concepts and processes involved. The main task in the lesson was to draw up an appointment schedule for a hair salon from a list of requests for appointments involving different hair treatments. This required students to use vocational knowledge about the time needed for each

treatment and considerations about appropriate business decisions, in conjunction with mathematics. The students produced individual schedules, using different methods and formats, but discussed their strategies and decisions freely with each other. The teacher supported and guided by asking students individually about their methods, assumptions and decisions. Finally, the teacher checked their progress with a longer-term integrated homework assignment in which students were using vocational knowledge, English and mathematics to produce a business plan for a new hairdressing salon.

Although the physical separation of the mathematics classroom from the students' vocational base was similar to the situation of the Public Services lesson, the key pedagogical features contrasted with those observed in David's lesson and were more closely aligned to the vocational session. Similar features were evidenced in a number of Functional Mathematics sessions and the cross-case analysis suggested the following key features of a more 'connected' functional mathematics classroom:

1. Teachers adopted pedagogies that made connections through context, classroom discourse and programme synchronization:

   - Using vocational situations as the context for mathematical problems. This was effective when the details of these scenarios were accurate and resembled situations that students had actually experienced;

   - Encouraging an integrated discourse about mathematics in students' lives by using informal conversations and interests as a basis for improvising discussions about applications of mathematics;

   - Synchronizing the Functional Mathematics scheme of work with the vocational programme to increase perceptions of relevance.

2. Teachers developed classroom cultures that were more in keeping with values of the surrounding vocational culture by:

   - Creating flatter social structures than those in traditional school mathematics classrooms;

   - Adopting a supportive, facilitating role;

   - Developing equitable relationships with students;

   - Using peer learning as a key learning strategy.

In cases where these features were present, our cross-case analysis suggested that students responded more positively to learning mathematics than in classrooms where the pedagogy and culture were more traditional. These features seemed to reduce the sense of disconnection between the mathematics classroom and the vocational programme and as a result their engagement with mathematics improved.

Discussion

For many of the students in this study, learning mathematics was perceived as separate from their vocational learning. However, when learning mathematics was connected to students' vocational development, values and culture then the subject generally became more relevant, meaningful and coherent. Although students retained a narrow focus on their vocational area (Hodgson & Spours, 2008) and only identified a limited utilitarian purpose for mathematics (Ernest, 2004), their acceptance of Functional Mathematics as a vocationally-relevant subject represented a shift in perspective that had a positive effect on their engagement with mathematics.

The students in this study were in a transition from school education to the workplace and were experiencing the tensions between formal, abstract academic learning and the development of vocational skills to achieve professional competence. As FitzSimons (1999) explains, mathematics in the workplace becomes a tool, in contrast to being the object of activity in mathematics classrooms. The transition from school to the workplace therefore involves changing students' perceptions of mathematics from object to tool, but this is a gradual process and not straightforward. In the interim period of being a 'trainee' in college students are caught between these two positions.

The mathematics learning of vocational students is situated in a complex socio-cultural environment, influenced by contrasting educational and vocational values and traditions. Although historical, social and cultural influences affect values generally in mathematics classrooms (Bishop et al., 1999), the co-existence of Functional Mathematics lessons within vocational programmes require students to change between cultures with typically dissonant values, unless cultural divisions can be bridged and values harmonized. In practice, students tend to adopt the values of their vocational area, as indicated by previous research (Colley et al., 2003) and the alternative value system, that frames much mathematics teaching, generates tensions. Some reconciliation of these different cultures is necessary to enable students to see learning mathematics as an integral and meaningful component of their vocational training.

Values relating to employment and adulthood were dominant in the general college culture and were also important to students. In some cases Functional Mathematics teachers created social structures that facilitated a more open and equitable classroom culture and this was better aligned to these values. Others embraced specific vocational values, such as teamwork for Public Services, in their teaching approaches. These adjustments to classroom culture provided a more coherent learning experience and helped stimulate student engagement.

In the observed vocational sessions, the role of the teacher was one of an occupational expert in a learning community similar to a 'community of practice' (Lave & Wenger, 1991; Wenger, 1999). In this social arrangement students were learning from the teacher's expertise and from one other by developing practical competencies coupled with technical knowledge. Functional Mathematics lessons involved a different learning process as students were neither aspiring to be mathematicians nor intending to be teachers of mathematics and therefore a 'community of practice' model was inappropriate.

The analysis suggests that the practices of a *connected* mathematics classroom in colleges can enable students to bridge some of these divisions by presenting mathematics as a subject that is not confined to the domain of academic knowledge but can also constitute vocationally-related skills. The pedagogy of the *connected* classroom in this study reflects some of the principles of embedding from previous research (Eldred, 2005; Roberts et al., 2005) but it also highlights the importance of shared values and compatible cultures in mathematics classrooms for vocational students. Bridges between different practices, of vocational learning and mathematics teaching, were constructed using key points in these separate discourses to make connections (Evans, 1999). These connections enabled a form of 'boundary crossing' that reconciled some of the conflicts for students between their vocational training and their learning of mathematics. Although the learning processes for mathematics and vocational skills retains some fundamental differences, these approaches suggest ways in which greater coherence and better engagement can be brought into the student experience of learning mathematics in vocational education.

Conclusions

The academic-vocational divisions and tensions of the English education system were evident in the student experience through contrasts in the pedagogy and purpose of learning in vocational and mathematics sessions but there were also important differences in the social structures, culture and values within the two separate learning environments. For students in transition from school to the workplace, the vocational training phase is characterised by changing values and shifting perspectives as students become more orientated towards employment. Bridging the divisions and providing a coherent, meaningful experience of mathematics learning for vocational students requires an understanding of this transition, a non-traditional approach to mathematics teaching and a classroom culture that reflects the values of the surrounding environment that are important to students. The effects of these features within the classroom were significant for students in the study and suggest aspects of teaching in a vocational environment that need to be considered seriously alongside general and subject-specific pedagogy.

In the light of recent policy changes in England it seems that the move towards the more knowledge-based, academic GCSE mathematics qualification rather than a 'functional' curriculum is likely to create a greater distance between mathematics and vocational learning for students. Further research is

needed to ascertain the actual effects of these policy changes on the dispositions and attainment of students who are required to re-sit GCSE mathematics courses but there are clear indications in this study that addressing the cultural divisions between mathematics and vocational learning is an important factor in creating a meaningful and successful experience for students. These findings have implications for the training of mathematics teachers for Further Education. They also raise questions for policy-makers for whom the achievement of an academic minimum standard in mathematics is privileged over engaging students in a meaningful experience that prepares them for the workplace.

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Appendix I

Table 1.
Student Views on What College is Like

QUESTION B: What is college like? (compared to school)					
STATEMENT	SA	A	N	D	SD
There is more freedom than there is in school	45	42	9	6	1
It has been easy to make friends	32	48	23	0	0
I get on with the staff in college	25	58	16	4	0
You are treated better at college than school	26	50	21	6	0
College work is easier than school	8	32	38	21	4
The staff treat you like adults	23	51	15	10	2
My course is interesting	47	44	8	2	1
I like the subjects I do	32	57	10	4	0

Table 2.
Student Experiences of Mathematics in School

QUESTION D: When you did Math at school how did you feel?					
STATEMENT	AA	S	H	O	AN
I worked hard	13	34	31	14	9
It was difficult	9	29	28	28	8
I got distracted	22	31	16	29	4
I liked Math	9	20	14	20	38
I felt stressed	15	20	19	29	19
I was bored	21	20	26	27	7
I liked the teacher	11	23	19	13	36
I felt confident	4	22	21	38	17
It was interesting	2	14	18	34	34
I understood it	6	30	27	24	15
It was confusing	14	21	23	38	6
I could have done better	34	30	17	19	0

Adult Learners and Mathematics Learning Support *

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Abstract

The provision of some level of Mathematics Learning Support (MLS) is now standard in the majority of Higher Education Institutions in Ireland, the UK, and in many other countries. This provision is, in part, a response to the large numbers of students entering Higher Education who do not have the mathematical skills required and this cohort includes a significant number of adult learners. Research indicates that these students have different motivations and approaches to learning than traditional age learners. This paper considers the analysis of a large scale student evaluation of Mathematics Learning Support in Ireland. In particular, it presents the responses and engagement levels of adult learners and compares these to those of traditional students. The findings are key to ensuring best practice in the provision of MLS for the wide variety of students who engage with it.

Keywords: adult learners, engagement, evaluation, mathematics learning support

Introduction

The availability of some form of Mathematics Learning Support (MLS) is now what students can expect to find in the majority of Higher Education Institutions (HEIs) in Ireland and the UK. MLS is also available in HEIs internationally, for example in Switzerland, Canada and Australia (Gill et al., 2008; Perkin et al., 2012). MLS has been defined as a facility offered to students which is surplus to their traditional lectures and tutorials, the purpose of which is to offer non-judgemental and non-threatening one-to-one support with mathematics (Ní Fhloinn, 2007; Lawson et al., 2003; Elliot and Johnson, 1994).

The main reason for the establishment and significant growth of MLS was to tackle the well documented 'Mathematics Problem'. One of the ways O'Donoghue (2004) defines the 'Mathematics Problem' refers to the mathematical preparedness of incoming students in terms of their mathematical

shortcomings or deficiencies at the university interface. Significant numbers of students entering HEIs are deemed at-risk of failing or dropping out because they do not appear to be appropriately prepared for mathematics in HE and often exhibit very weak mathematical backgrounds. This 'Mathematics Problem' is common place in HEIs in Ireland, the UK and internationally (Gill et al., 2010; Lawson et al., 2012). These at-risk students are the main target of MLS.

One benefit of the economic downturn has been the welcome increase in adult learners returning to HE (Golding and O'Donoghue, 2005). In the Dublin Institute of Technology (DIT), adult learners constituted one fifth of the attendants at the Mathematics Learning Support Centre (MLSC) in its opening year (Ní Fhloinn, 2007). In 2012 adult learners accounted for 15.3% of full time students enrolled in HE in Ireland and 21% of full and part time students. Faulkner et al. (2010) stated that the presence of so many adult learners is one contributing factor to the increased numbers of at-risk students in first year courses.

In order to establish best practice in the successful provision of MLS, it is essential that it is comprehensively evaluated on a regular basis (Matthews et al., 2012). For example, quantitative research suggests that appropriate engagement with MLS can have a positive impact on student retention and progression (Lee et al., 2008; Mac an Bhaird et al., 2009). One of the initial aims of the Irish Mathematics Learning Support Network (IMLSN), which was established in 2009, was to conduct a large scale survey of student opinion on MLS. A full report on this survey was published in November 2014, and is available from http://epistem.ie/wp-content/uploads/2015/04/IMLSN-Report-16102014Final.pdf. There is some overlap between the final report and results presented in this paper (submitted June 2014).

Given the increasing proportion of adult learners in mathematics in first year courses, it was considered key that they should be identifiable in the survey so that their responses regarding the evaluation of MLS could be studied in detail. A chi-square test for independence carried out on the data collected indicated a statistically significant association existed between type of student (i.e. adult learners or traditional learners) and whether a student used MLS ($p<0.001$), thus demonstrating that adult learners were more likely to seek support than traditional age learners. Further investigation however demonstrated that 38% of the adult learners surveyed never accessed MLS in their institutions. The authors decided to investigate the underlying reasons behind these findings further.

The main research questions we are trying to address are:

1. What are the motivational factors of adult learners who seek MLS?
2. Why do some adult learners of mathematics not seek MLS?

Literature review

There is a concern that a lack of preparation in mathematics can lead to increased failure rates and low self esteem (Symonds et al., 2007) in HEIs. Aligned with that is a worry of impeding students in the study of other disciplines, e.g. engineering, science (Pell and Croft, 2008; Gill, 2006). Many students arrive in their HEI having chosen mathematics-intensive courses unbeknownst to themselves (FitzSimons and Godden, 2000). Most degree programmes, even non-specialist mathematics degrees, contain some mathematics and/or statistics component, as prospective employers require graduates to be proficient in mathematics, with some even setting numeracy tests as part of their selection process (Lawson et al., 2003). The mismatch between the knowledge of many students and the expectations of HEI teachers is one contributory factor to the problem and this mismatch arises partly through the increase in diversity of the backgrounds of students (Lawson et al., 2003; Faulkner et al., 2010). Diversity in the standards of teaching and class size in HEIs tend to exacerbate the situation (Lawson et al., 2003; Gill, 2006).

One of the key responses to the 'Mathematics Problem' was the opening of Mathematics Learning Support Centres (MLSCs) to attempt to deal with the mathematical shortcomings of students (Pell and Croft, 2008; Gill, 2006). In 2004 in the UK it was reported that 62.3% of 106 surveyed universities offered some form of MLS (Pell and Croft, 2008, p. 168). In 2012, this number had jumped to 85% (Perkin et al., 2012). In 2008, an audit carried out by the Regional Centre for Excellence in

Mathematics Teaching and Learning (CEMTL) in Ireland demonstrated that 13 out of 20 HEIs provided MLS in some form (Gill et al., 2008). Seven years later, it is believed that this number is much higher. Most MLSCs are committed to servicing the needs of traditional and non-traditional (i.e. international and adult learners) students (Ní Fhloinn, 2007; Gill and O'Donoghue 2006). Carmody and Wood (2005) reported on the benefits of a drop-in MLSC for easing the transition to HE for first-year students. The drop-in centre caters for students from all faculties and has become a meeting place for collaborative learning. Tutors use a variety of teaching methods and resources, which is easier to do in a one-to-one situation than in front of a large class. Engagement with MLS has been shown (through mostly quantitative research) to impact positively on mathematics performance and grades and retention (Burke et al., 2012; Mac an Bhaird et al., 2009; Pell and Croft, 2008; Symonds et al., 2007). Pell and Croft (2008) state that while MLS is provided first and foremost for 'at-risk' students, it is more often the case that users tend to be high achievers working to attain high grades, a view supported by Mac an Bhaird et al. (2009) who have also shown that many 'at-risk' students still do not engage with MLS.

An Adult Learner, or Mature Student, is classified in the Republic of Ireland as a student that is 23 years of age or older on 1st January of the year of registration to HE (Ní Fhloinn, 2007). Entry for adult learners who have not got the minimum requirement for entry to their chosen course of study is usually gained via interview and is based on a number of factors including life experience and motivation, in addition to prior qualifications. Faulkner et al. (2010) studied the student profile in service mathematics programmes at the University of Limerick (UL) since diagnostic testing began there in 1997. The increase in adult learners of mathematics in these modules was quite pronounced. In 1997 there was one registered in Science and Technology Mathematics, two of the biggest service mathematics modules provided by this university; in 2008, there were at least 55 adult learners. This statistic is supported by Gill (2010) who states that in 2009/10, adult learners in UL constituted 14% of the entire cohort, an increase of 49% on the previous year. In 1997, 30% of students in one service mathematics modules at UL were deemed to be at-risk. Fast forward to 2014 and 66% of students in the same module are categorised as at-risk.

Adult learners of mathematics who return to education constitute a heterogeneous cohort. For example, participants on the 'Head Start Maths' bridging programme at UL range from 23 to over 45 years of age. A significant number of the students on the programme in 2008 had not studied mathematics in any formal sense for up to 20 years and 30% of participants had not taken the Leaving Certificate (LC) examination (Gill, 2010). The LC is the terminal examination taken by pupils at the end of secondary school in Ireland. Mathematics is compulsory for students and can be taken at three levels: Higher (HL), Ordinary (OL) and Foundation (FL). In DIT, Ní Fhloinn (2007) outlines how adult learners fall into the full-time, part-time or apprenticeship categories, with each type of student presenting with different characteristics and issues relating to their preparation, their approach to learning mathematics and confidence issues. It can be very difficult for students to catch up with forgotten fundamentals and keep up with current studies simultaneously (Gill, 2010; Lawson et al., 2003).

Diez-Palomar et al. (2005) and O'Donoghue (2000) acknowledge the difference between adult learners of mathematics and traditional learners. Adult learners carry with them an abundance of experiences that need to be considered in pedagogical practices. This view is supported by Tusting and Barton (2003) who add that adult learners have different motivations for studying than traditional learners and are more inclined to be autonomous and reflective learners. The decision to return to education has generally been their own decision and a deliberate one (FitzSimons and Godden, 2000). Though adult learners may lack confidence in their own abilities, they tend to be highly motivated (Ní Fhloinn, 2007; FitzSimons and Godden, 2000). Traditional lectures and assessments are not conducive to learning for many adult learners (Gordon, 1993 cited in FitzSimons and Godden, 2000) so many rely on MLSCs for support. In 2009/10 adult learners of mathematics at UL constituted 54% of the attendance at the drop in centre, even though they represented just 14% of the entire student population (Gill, 2010).

While the importance of research in the teaching and learning of mathematics among adult learners has been duly recognised in recent years (Coben, 2003) it remains an 'under theorised and under

researched' area (Galligan and Taylor, 2008, p. 99). Furthermore, research conducted on the teaching and learning within MLSCs is sparse (Galligan and Taylor, 2008).

Methodology

The IMLSN was established in 2009, and its guiding principles are similar, on a smaller scale, to the leading experts in the provision of MLS, the **sigma** (The Centre of Excellence in Mathematics and Statistics Support) network (http://sigma-network.ac.uk/) based in England and Wales. The IMLSN aims to support individuals and HEIs involved in the provision of MLS in Ireland. Once set up, the network decided it should promote the benefits of MLS to both staff and students on an institutional, national and international basis and agreed that a student survey was the best approach initially. The IMLSN asked the panel of researchers listed on this paper to undertake this student survey.

Student questionnaires are commonly used in the evaluation of MLS services (Lawson et al., 2003) in individual HEIs, so it was decided to create a student survey that could be used in all HEIs which provide MLS. HEIs who already distributed questionnaires on MLS were invited to submit them to the committee; these were amalgamated and a communal questionnaire was formed as a result. This questionnaire was piloted in 4 HEIs with 100 students and subsequently refined based on analysis of the findings and expert statistical advice.

The resulting questionnaire (See Appendix I) had 17 questions, a combination of open questions and questions which required a response on a 5-point Likert scale. There were three main sections: Section A determined the students' backgrounds; Section B focused on users of MLS; and Section C focused on non-users of MLS. First year service mathematics classes have the largest percentage of at-risk students and are the main target of MLS in terms of student retention and progression, so it was decided to issue the questionnaire to these cohorts only. Service mathematics refers to users of mathematics (e.g. engineering, science, business), rather than mathematics specialists (e.g. pure or applied mathematicians) (Burke et al., 2012). Evaluation sheets are usually distributed within MLSCs but this can lead to bias as users already rate the MLSC to some extent if they attend it (Lawson et al., 2003). With this in mind, it was decided that the questionnaire should be issued in mathematics lectures to get a blend of user and non-user feedback and to reduce bias. The questionnaires were anonymous and there were no identifying characteristics. The questionnaire was issued to members of staff involved in the provision of MLS in HEIs in Ireland and they were asked to distribute paper copies in first year service mathematics lectures during the second semester of the 2010-11 academic year.

The HEIs surveyed were Universities and Institutes of Technology (IoTs), and these have different and complementary roles and missions within HE in Ireland. At undergraduate level Universities focus on Level 8 (Honours Degree programmes), and IoTs emphasise career-focused HE offering Level 8 programmes but also Level 7 (Ordinary Degrees) and Level 6 (Higher Certificates) programmes. IoTs also have a larger proportion of adult learners and students from disadvantaged areas and are stronger than the Universities in part-time and flexible provision (http://www.hea.ie/en/node/981). In the IoTs that participated in the survey, the ratio of Level 8: 7: 6 students was 49:39:12% which is similar to the 53:38:9% proportion of Level 8: 7: 6 students in IoTs nationally in the 2011-12 academic year. There are 7 universities and 13 IOTs in the Republic of Ireland. All institutions were invited to take part in the study by contributing their current evaluation methods and/or distributing the resulting survey to their students. 5 universities and 4 IOTs volunteered to take part, culminating in 1633 responses, 13.5% of whom were adult learners.

Two graduate students were hired to input the data into SPSS, and SPSS was also used to analyse the quantitative data. NVivo was used to analyse the qualitative data. A general inductive approach was used to analyse the data guided by the specific research questions (Thomas, 2003). Data was read and analysed by two researchers independently, one from this panel of researchers and an external person to identify emerging themes. Further details on the analysis to date for all respondents (traditional students and adult learners combined) can be found in (Mac an Bhaird et al., 2013 and Ní Fhloinn et al., 2014).

Results

In Section A of the survey questions were asked which focused on students' backgrounds. Of the 1633 respondents, there were 221 (13.5%) adult learners, 73% of these were male and 91% were full-time students. In terms of students' mathematical background, they were given the 4 options outlined in Table 1. Generally, a minimum of OL mathematics would be needed for most service mathematics courses in HEIs and this is reflected among respondents (18 of the 1546 students who provided their LC results in the survey had studied mathematics at FL). If they had not taken the LC, then they could select the Other option.

Table 1.
Mathematical Backgrounds of Adult and Traditional Learner Respondents

Higher Level LC	Ordinary Level LC	Foundation Level LC	Other/N/A	Missing	Total
37.1% (516)	60.7% (843)	0.6% (9)	0.9% (12)	0.6% (9)	Traditional Learners (1389) *
9% (20)	67.4% (149)	4.1 % (9)	14.1% (31)	5.4% (12)	Adult Learners (221)

*Out of 1633 responses, 1389 identified themselves as not being adult learners, 1 was an exchange student and 22 did not tick any box.

A lower percentage of adult learners (than of the traditional learner respondents) had taken HL, and higher percentages (compared with the traditional age students) in the remaining three categories, with the majority studying mathematics at OL.

When the breakdown of the disciplines that students were in was considered, we found, for most discipline areas, the proportion of adult learners was in line with the proportions of the traditional learner respondents, see Table 2.

Table 2.
Degree Programmes of Adult Learners and of Overall Survey Respondents

Subject	No. of Adult Learners	%	No. of Traditional Learners	%
Science	80	36.2	494	35.6
Engineering	50	22.6	183	13.2
Business	55	24.9	418	30.1
Arts	7	3.2	58	4.2
Education	6	2.7	83	6
Computing	23	10.4	148	10.7
Health Sciences	0	0	4	0.3
Total	221	100.0	1388*	100.0

*1 exchange student and 12 missing data

Section B of the questionnaire focused on MLS users. The majority of adult learners 136 (61.5%) availed of MLS, compared to only 32.2% of traditional learners. A Chi-Square Test for independence indicated a statistically significant association exists (p<0.001) between type of student (i.e. Adult or traditional learner) and whether a student uses MLS: adult learners were more likely to seek MLS than traditional learners. In terms of gender 68.3% of female adult learners compared to 43% of female traditional learners used MLS, and 59.4% of male adult learners in comparison to 23.3% of male traditional learners availed of MLS.

The mathematical backgrounds of both users and non-users of MLS among the adult learner sample were very similar, and the percentage breakdown was close to that of the adult learner population (See Table 1). When we considered subject discipline, the proportions of adult learners using MLS was very similar to the proportions of overall adult learners in each subject discipline (See Table 2).

Students who availed of MLS were asked, in an open–ended question, to comment on why they first decided to use MLS. There were 577 comments from attendees which were coded using GIA and the majority fell into 6 main categories as outlined in Table 3. This table contains comments from 122 of the 136 adult learners who responded.

A comparison of the frequency of responses in each category given by adult learners compared with traditional learners provides some interesting differences. The frequency of responses from adult learners showed they are much more likely to make comments indicating that they:

- look for help as they have a long time away or suggesting poor confidence in their mathematical ability (19.67% as against 3.96% for traditional learners),

- seek general extra help (38.52% as against 15.17% for traditional learners),

- are struggling (9.02% as against 3.74% for traditional learners).

Table 3.
Frequency of Adult Learner Reasons for Using MLS

Categories of comments	Frequency of comments (n=122)	Sample comments
Extra help	38.52%	"Needed help with maths", "I had gone to the tutorials and still had trouble with a particular area", "I wanted help with a Mathematics Problem and to understand where I was going wrong", "Because the pace of the main lectures were too fast and I wasn't keeping up", "I had to catch up on missed lectures"
Background/Ability: Comment about being away from Maths for a while prior to entry (from mature students) or comment suggesting poor confidence in maths ability	19.67%	"Hadn't done maths in ages so I needed extra help", "Because I haven't studied maths in ten years and really felt quite daunted by the thoughts of returning to study maths", "Coming back to study after a long break, needed all the help at hand!", "Because I am not great at maths"
Assignments/Exams: Looking for help with specific aspect of coursework assessment during the semester (upcoming test, assignment) or attending for revision or preparation for end of term examinations	13.93%	"Struggling with maths assignments", "I was stuck on understanding a part of an assignment and was spending a lot of time trying to figure it out", "To help with revision"
Struggling	11.48%	"I was struggling with the subject", "Was lost with maths"
Improve Understanding: Positive comments about attending to try to improve or gain better understanding	5.74%	"Because I thought it will be a great idea to use drop-in clinic if I want to get good grades"

In contrast, the frequency of responses from adult learners shows they are much less likely to make comments indicating that they:

- seek help specifically to get assistance with particular coursework assessment or revision for tests (13.93% as against 47.47% for traditional learners)

- attend MLS to improve or gain better understanding (5.74% as against 18.24% for traditional learners).

- state they find mathematics difficult (2.46% as against 11.43% for traditional learners).

MLS users were asked to rate, on a 5-point Likert scale, the specific services available in their HEI and they were also given the opportunity to comment. The main support offered was a drop-in centre, so we focus on that support in this paper. The distribution of ratings and responses from adult learners for the other services (e.g. ICT supports, workshops, support tutorials) are in line with that of the overall cohort.

All nine HEIs had a drop-in centre and 519 users rated them. 119 were adult learners and 89% of these rate it as worthwhile. There were 244 additional comments, 57 from adult learners and coding of responses placed them into the following three main categories (Table 4):

Table 4.
Adult Learner Rating of MLS Services

Categories of comments	Frequency of comments (n=57)	Sample comments
Satisfaction with service	38.5%	"Very helpful – I am even starting to enjoy maths now", "Would not have a clue what I was doing if it was not for support"
Physical Resources	40.4%	"Class size was small for the amount of students", "If there were more opening hours and people available as it is very busy", "Sometimes a long waiting time; too busy"
Quality of Tutors	17.3%	"Always as helpful as they can be with the exception of one of the tutors who tends to be very rude and arrogant".

20 (38.5%) responses related to satisfaction levels with the service provided, 19 of which were positive. 23 (40.4%) comments related to the physical resources, including staff and contact hours of the centres. Without exception, all comments stated that all of the above should be extended. 9 (17.3%) related to the quality of tutors; 5 positive, 1 negative and 3 which were positive and negative simultaneously.

In Questions 11-15, MLS users were asked about their perception of the impact of MLS on various aspects of their education, the questions had a 5-point Likert scale and they could also comment on their answers. Students were asked to rate the impact MLS had on their confidence. 539 users responded, 125 were adult learners and 66.4% of these rated the impact as helpful in comparison to 52.3% of traditional learner users. There were 106 additional comments, 21 from adult learners with 20 of these positive, "It has helped me a lot. I don't need to struggle alone to figure out things that I don't understand", "Still find it difficult but have a better understanding of maths". For traditional learners, approximately 71% of comments were positive.

Students were also asked if MLS had impacted on their mathematics performance in tests or examinations to date. There were 534 responses, 122 from adult learners and 61.5% of these stated that it had an impact, in comparison to 52.8% of traditional learner users. There were 103 additional comments, 21 by adult learners, 16 of which were positive (93% of comments from traditional learners were positive), for example: "I would have failed if the extra help had not been there".

Students were asked to rate how MLS had helped them cope with the mathematical demands of their courses. There were 527 responses, 120 from adult learners and 72% of these indicated that MLS had been helpful in comparison to 62.5% of traditional users. There were 55 additional comments, 14 from

adult learners, 12 of which were positive, for example "It has been a huge help", "Wouldn't be able to do maths without all the extra services and wouldn't have a hope of passing the year". One of the (two) negative comments stated "Some of the tutors in the centre might be good at understanding maths but not good at teaching it".

In Question 11 students were asked if they had ever considered dropping out of their studies for mathematics-related reasons. 128 of the 136 adult learners answered this question with 25 (19.5%) stating that they did consider dropping out, this is a smaller proportion to that of the traditional student population (22.8%). Question 12 asked (those who answered yes to Question 11) if MLS had been a factor in them not dropping out. 22 of the eligible 25 adult learners answered and 17 (77%) of these stated that MLS was an influencing factor in their decision not to drop out (compared to 54.3% of the traditional learner cohort). Additional comments included: "Greatly. It has given me the confidence to turn maths as my worst subject into one of my best" and "Encouraged me to trust that my worries were normal and that practice would improve me". 8 students left comments stating that they never considered dropping out because of the MLS that was available to them, "Never felt the need because of the support provided" and "No, but did worry about failing maths before using these facilities".

Section C of the survey focused on students who had not availed of MLS. 85 (38.5% of) adult learners (compared with 67.8% of traditional learners) stated that they did not use the MLS facilities provided in their institution. In Question 16, non-attendees were asked to select from 7 fixed options, as to why they did not avail of MLS. For adult learners, the frequency of response in each category is interesting when compared with the traditional 941 students who did not use MLS, see Table 5 (note that students selected more than category).

Table 5.
Frequency of Reasons for Not Using MLS Between Adult Learners and Traditional Students

Category of response	% of Adult Learners who did not avail of MLS (n=85)	% of traditional students who did not avail of MLS (n=941)*
I do not need help with Maths	43.53%	49.4%
The times do not suit me	43.53%	27.8%
I did not know where it was	5.88%	18.6%
I hate Maths	3.53%	16%
Other	15.29%	12.6%
I was afraid or embarrassed to go	8.24%	11.8%
I never heard of the MLSC	15.29%	7.8%

*1609 answered the question 'Have you used any of the Maths Learning Support Centre's services?',583 answered 'yes', 1026 answered 'no', 24 gave no reply.

In terms of individual respondents, it is worth noting that of the 85 adult learners who did not avail of MLS, 43.53% of these stated that they did not need help. In comparison, for the 941 (67.8%) traditional learners who did avail of MLS, 49.4% of these stated that they not need help. We can see in Table 5 that a larger percentage of responses from adult learners stated that the times did not suit and that they had not heard of the MLSC. The proportions of adult learners responding that they hated mathematics, did not know where MLS was or were afraid or embarrassed to go, were much lower than in the traditional cohort.

There was an opportunity to provide additional comments on responses given to Question 16 and 34 adult learners did so. 20 comments stated that they did not need help or were able to work it out by themselves; 8 comments stated that the session timings did not suit them due to timetable or living circumstances; 2 stated that they never heard of the MLSC services; 2 comments related to a reluctance to attend: "Just felt a bit uncomfortable; felt the questions I had may seem a bit irrelevant". These responses were consistent with overall student comments.

In Question 17, non-users of MLS were asked to comment on what would encourage them to use the MLS facilities. The responses were coded into categories using GIA and Table 6 below gives the breakdown of responses from the 41 adult learners who answered. Compared with the traditional

student responses, adult learners were more likely to comment that they would access MLS if they needed. They were less likely to comment on resources/location or the need for student feedback or advice as reasons that would encourage them to engage with MLS. No adult learners mentioned examinations or results as a prompt for them to access MLS.

Table 6.
Frequency of Comments from Adult Learners who are Non-Users of MLS about What Would Encourage Them to Engage with MLS

Category	% of Responses (n=41)
Go if needed	46.34%
Results/Exams	0%
Better times	19.51%
More Information	19.51%
Resources/Location	4.88%
Advised to go	2.44%
Student Feedback	2.44%
Miscellaneous	4.88%

Discussion and conclusion

In this paper we have considered the data concerning adult learners in our large-scale student evaluation of MLS. We also compared, where possible, these results with from the traditional learners. Our two main research questions were:

1. What are the motivational factors of adult learners who seek mathematics learning support (MLS)?

2. Why do some adult learners of mathematics not seek MLS?

When we considered the backgrounds of the respondents, we did not find a significant difference between adult learners and the traditional learner cohort in terms of the disciplines that they were studying. This will be investigated further in the next stage of our analysis when we consider the breakdown of results in terms of the individual institutions that respondents attended. However, as one would expect, adult learners did present with a wider range of mathematical backgrounds than the traditional cohort, with a smaller proportion taking HL and a higher percentage taking OL. This is consistent with research elsewhere, e.g. Gill (2010).

When students who engaged with MLS were considered, there was a statistically significant association (Chi-Squared Test, $p<0.001$) between student type (i.e. adult learners or traditional) and whether a student uses MLS, demonstrating that adult learners are more likely to seek support than traditional learners. This supports other research, e.g. Ní Fhloinn (2007) who states that adult learners in DIT seek support much earlier than traditional learners, even as early as the first day of term. However, in our study, we found no significant difference in the mathematical backgrounds of adult learner users and non-users of MLS.

Partial answers to our first research question are provided when the reasons why students engaged with MLS were investigated. Analysis suggests that adult learners in our study were more likely than traditional students to mention the following reasons for engaging: having been a long time away from education; poor confidence in their mathematical ability; seeking general extra help; struggling with mathematics. In contrast, adult learners were much less likely than traditional students to mention the following reasons: to get help with specific coursework assessment or as revision for tests; to improve or gain better understanding; to state they find mathematics difficult. Being an adult learner, having not studied mathematics in any formal sense for a long time lends itself to having gaps in knowledge due to forgotten or perhaps never learned material. Lawson (2008) states that some students avoid support due to a fear of embarrassment or feeling that they just have too many mathematical problems to deal with. This gap in knowledge appears to act as an impetus rather than an obstacle for the adult learners in our study to engage with support "As I have been out of the education system for many

years I felt I needed the extra support". These adult learners were motivated to engage because of their worry about gaps in their mathematical knowledge and the length of time they had been away from studying mathematics "As a mature student I needed a refresher". Wolfgang and Dowling (1981) may partially explain this finding as they maintain that traditional and adult learners have different motivations and approaches to study. Safford (1994, p. 50) supports this stating that while adult learners may carry 'intellectual baggage', they are generally self-directed and making the decision to return to education implies a motivation for change and growth.

A significantly smaller proportion of adult learners did not avail of MLS when compared to the overall cohort. In terms of our second research question, we considered the reasons given by students for non-engagement with MLS. According to Ashcraft and Moore (2009) avoidance is often the consequence of mathematically anxious students. Bibby (2002) reports that math anxiety and shame of own mathematics ability are reasons that students fail to seek help with mathematics. In a study carried out by Grehan et al (2011, p. 79) at NUI Maynooth, the reasons divulged for lack of engagement with MLS included 'fear; lack of personal motivation; the anonymity of large classes; and to a lesser extent the lack of awareness of support services'. Symonds et al. (2008) list a fear of embarrassment and a lack of information regarding the whereabouts of the mathematics support as reasons why students do not engage. Our findings largely contrast with those just mentioned. The largest proportion of responses from both adult learners and the overall cohort who did not engage with MLS indicated that they simply did not need to: "Good service for students – just didn't need to avail of it"; "I would definitely find time to attend if I needed to". It is reassuring that many of those who do not utilise the resources provided simply do not feel the need. 5.88% of adult learners who had not engaged with MLS stated that they did not know where it was and 15.29% had not heard of the support. 8% stated that they were afraid or embarrassed to go "Just felt a bit uncomfortable, felt the questions I had may seem a bit irrelevant". As we discussed earlier, fear and embarrassment were more of a motivation to attend rather than not attend MLS. 43.53% of adult learners who did not engage with MLS stated that the times were unsuitable. These statistics are enlightening as they have implications for MLS practitioners in terms of marketing and advertisement of services and extension or alteration of opening hours to maximise participation for those who require additional help.

Overall, respondents were very positive about the MLS experience they received in their institution, with adult learners especially so, e.g. users of MLS reported increased confidence in the mathematical abilities and finding it easier to cope with the mathematical demands of the courses "I've had a fear of maths all my life so with MLC help I've become more confident". It is clear from the comments that MLS provides a mathematical lifeline, so to speak, for many adult learners: "I would be seriously lost without the MSC and the extra maths classes ran. Now I actually like maths"; "Excellent and I credit the help I receive here to me passing all my maths tests so far".

Many of the comments highlighted the important role of MLS tutors. Lawson (2008) states that students attend MLSCs precisely because they offer emotional and MLS to students who suffer from mathematics anxiety. FitzSimons and Godden (2000), and Safford (1994) recommend the provision of this warm supportive environment in which individual needs are met and adult learners of mathematics can thrive. The quality of staff is crucial to the success of MLS (Lawson, et al., 2003) and in particular in relation to the education of adult learners (FitzSimons & Godden, 2000). Gill (2006) states that the one-to-one attention students receive in MLSCs is most highly favoured. Some of the responses in this study referred to how they preferred the teaching approach used in the MLSCs to those in their regular tutorials "People in the MLSC explain the questions or doubts you have the way the people in the tutorials should".

However, Lawson et al. (2003) states that not everyone will make a good MLS tutor and this is reflected by the small number of negative comments about certain MLS tutors, e.g. "Possibly some training in social skills for some of the tutors". Benn (1994) encourages teachers to tread carefully when dealing with Mature Students of mathematics as it will influence how students perceive the subject. It is in the nature of MLS evaluation that both positive and negative comments can be used constructively. To this end, the IMLSN is in the process of developing MLS tutor training materials which will be used in the academic year 2015/16 to help ensure best practice in the recruitment and training of tutors across all institutions in Ireland. There were some other negative comments, e.g. in

relation to the timing of the drop in centre or classes, the volume of students in attendance and hence the lack of one-to-one attention at busy times: "It's sometimes very crowded and the instructors cannot get to you", "Sometimes the wait for assistance is 30-45 minutes". These findings resonate with those of Lawson et al. (2003) who state that MLSCs are inclined to be very busy at certain times, such as at examination time, and there will be waiting times as a result. Again, these comments were not standard across the survey and will be of more relevance to the individual institutions when further analysis is presented.

It is very difficult to claim that MLS is responsible for increases in retention or student success rates in mathematics (Lawson et al., 2003). Mac an Bhaird et al (2009) tell us that we cannot take full credit as a number of factors are in play when it comes to student progress such as motivation etc. However, the findings from this study indicate a high level of satisfaction with the services provided by the MLSCs throughout Ireland, and many adult learners indicated that MLSCs are responsible for their not dropping out of their studies. "It was a very valuable experience, whereby without it I would have certainly failed."

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Appendix I: Sample mathematics learning support survey

This appendix contains a sample from one institution of the questionnaire used. All questions with the exception of Question 10 were identical in all HEIs in which the questionnaire was distributed. The structure of Question 10 was the same as the sample shown here but the list of supports and names used to describe the supports which the students were given in Question 10 was localised to take account of the specific supports offered in that HEI and the names they are given there. The only other variation in the questionnaire was the localisation of the name given to MLS in that HEI – for example in one HEI the provider of MLS is known as the MLSC (Mathematics Learning Support Centre), in another it is known as the MLC (Mathematics Learning Centre) and in another it is known to the students as CELT Mathematics Services.

Mathematics Learning Support Survey

We are looking for your feedback on the Mathematics Learning Support Centre (MLSC) and its services. This evaluation is designed to help us to improve the MSC for you and other students. Even if you have not used the MLSC's services, your feedback is important.

Section A

1. Degree Programme:
2. Year: **Certificate 1st year   2nd year   3rd year   4th year Postgrad**
   Student Category: **Full-time   Part-time**
3. Gender: **Male   Female**
4. Leaving Certificate Mathematics Level (if applicable):
   Higher   Ordinary   Foundation   Other
5. Leaving Certificate Mathematics Grade (if applicable):
6. **Leaving Cert 1991 or before:** A   B   C   D   E Other
7. **1992 or after:**   A1 A2 B1 B2 B3 C1 C2 C3   D1 D2 D3
   Other
8. If you started off doing Leaving Certificate Higher Level Mathematics, but changed to Ordinary Level, roughly when did that happen? (Please circle)
   Before Christmas in 5th year   Before the end of 5th year
   Before Christmas in 6th year   After the Mocks in 6th year
   N/A
9. Are you registered as a mature student? **Yes   No**
10. Have you used any of the Maths Learning Support Centre's services (drop-in centre, support workshops, online courses)?
    Yes   No

If YES, please proceed to Section B.
If NO, please proceed to Section C.

Section B (Students who used the MLSC)

11. Why did you first decide to use the MLSC or its services?
12. Being as honest as you can, rate the following services that you have used below on a scale of 1 to 5 where 1=Not at all Worthwhile and 5=Extremely Worthwhile
    Drop-In Centre
    1   2   3   4   5   N/A
Comments/Suggestions:
    Online Courses
    1   2   3   4   5   N/A
Comments/Suggestions:
    Workshops
    1   2   3   4   5   N/A
Comments/Suggestions:
    1. Did you ever consider dropping out of your course/college because of mathematical difficulties?
    Yes   No
Comments:
13. If yes, has the MLSC influenced your decision not to drop out?

Yes   No

Comments:

14. Rate how the MLSC has helped your confidence in maths on a scale of 1 to 5 where 1=Not at all

15. Helpful and 5=Extremely Helpful

1   2   3   4   5

Comments:

16. 14. Rate how the MLSC has impacted on your maths performance (in exams/tests) so far on a scale of 1 to 5 where 1=No impact at all and 5=Has had a large impact

1   2   3   4   5

Comments:

17. Having used some of the MLSC's services, rate on a scale of 1 to 5 how you feel the MLSC has helped you cope with the mathematical demands of your course where 1=No help at all and 5=Has been a huge help

1   2   3   4   5

Comments:

Any other comments or suggestions about the MLSC Services would be very valuable!

Section C (Students who did not use the MLSC)

18. If you did not use the MLSC, why not? Tick as many reasons as apply:
    - ☐ I do not need help with Maths
    - ☐ I never heard of the Mathematics Learning Support Centre
    - ☐ I did not know where it was
    - ☐ The times do not suit me
    - ☐ I was afraid or embarrassed to go
    - ☐ I hate Maths
    - ☐ Other (please specify):
    - ☐ Comments:

19. What would encourage you to use the MLSC and its services if you needed to?

Any other comments or suggestions about the MLSC Services would be very valuable!

Let's Talk About Math against Symbol-Shock

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Abstract

About 1/3 of young adults between the ages of 16 and 20 are not literate enough in either reading advanced texts nor in reading comprehensive mathematical expressions with operators. As one of several explanations for this kind of 'illiteracy', we take into account that the languages of mathematics, physics, chemistry, etc. all contain unfamiliar symbols, or possible 'shock inducing elements', which we propose leads to a phenomenon called 'symbol-shock'. As well as having other symptoms 'symbol-shock' may provoke symptoms associated with frightening or freezing behavior. Shock symbols are defined as 'non-meaningful gaps' which cannot be interpreted or filled with any meaning. From our quasi-experimental study of consulting young adults, we propose several heuristics for prevention of symbol shock. We propose a set of self-management instructions to establish a cognitive and emotional meta-system to minimize 'symbol-shock'.

Keywords: symbol-shock, trauma, literacy, mathematics, meta-cognition

Observations

Literacy of mathematical terms with variables and function symbols is often not sufficient to succeed in an exam scenario. Among many factors influencing the performance in mathematical comprehension and problem solving (interest in mathematics, individual attitude to science, learning behavior, learning environment, etc.) we focused our study in particular on the confrontation of the symbolic language of mathematics. Previous studies by Kettler published in 1991 and 1998, showed that the combinations of algebraic mathematical symbols and operators may cause a cognitive and emotional behavior called 'symbol shock'. This behavioral response of being shocked appears in a similar manner to psychological testing with the Rorschach inkblot test where the sudden change of black and white testing cards to cards with colored inkblots reportedly caused disruption in behavior came to be known by Rorschach as 'color-shock'. In his early studies by Rorschach (Bohm, 1996) the subjects stopped interpreting the inkblots for several seconds which is interpreted as a dishabituation response in cognitive psychology.

Symbol-Shock as a Metacognition

For a basic understanding of the 'symbol-shock' phenomenon we refer to the research of metacognition in cognitive psychology (Flavell, 1979): 'Metacognitive experiences are any conscious cognitive or affective experiences that accompany and pertain to any intellectual enterprise. An example would be the sudden feeling that you do not understand something another person just said.' (p. 906).

Metacognitive sensations in the sense of Flavell may be conscious or 'unconscious' in the sense that it occurs automatically. This can be recognized on a behavioral level as:

- 'Overflying' (i.e. superficially scanning)
- Acting exclusively along schemas
- Looking for help
- Complete resignation

Effects of not responding to language with unknown symbols while being presented with mathematical formulas may be similar to that of 'freezing behavior' similar to other forms of reaction to stressful events and shock experiences. Freezing behavior is understood in biology and psychology as a form of reaction to trauma (Berceli, 2009), but the release of trauma or shock symptoms regarding symbol-shock will be another topic not explicitly addressed in this study.

A quasi-experimental setting

In order to study the experience of an eventual symbol-shock, we presented young adults with three photos from a publication by a Swiss photographer (Pol, 2014): we asked them to describe the meaning of the symbolic elements and mathematical operators in the expression written in the right hand corner of the blackboard in the following picture.
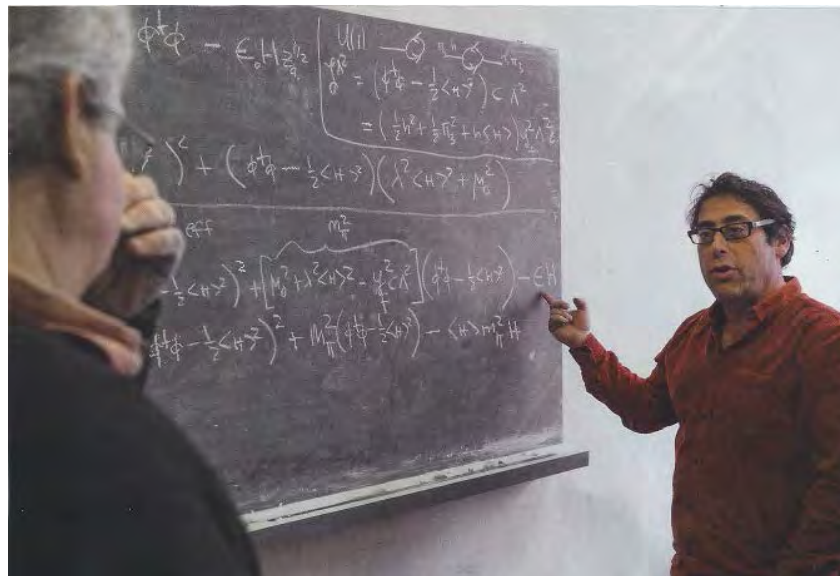
Figure 1. Discussing physicists at CERN, (copyright Pol 2014)

Method and Observations

With background in Socratic psychology and more specifically in 'Socratic consulting' (Hake, 1992) our team looked for metacognitive strategies used by the young adults, mostly skilled craft trainees, by asking questions such as: 'Describe what you see.' and 'What do these mathematical expressions on the black board mean to you?' and finally 'What do you think these expressions mean to the people in the pictures?'

The observed reactions of the trainees confronted with unfamiliar mathematical expression in this half-experimental setting were quite different. We qualified quite a few different answers as possible forms of 'shock-reactions':

- 'Seeing these kinds of formulas upsets me';
- 'Letters are for textbooks not for mathematics';
- 'Nobody can understand this stuff';
- 'May I leave the room?' etc.

Let's Talk About Math

Based on a Swiss reference textbook, 'Mathematik im Gespräch' (Discussing Mathematics, Liechti et. al. 1993), which navigates the teaching process of mathematics, we encouraged the trainees to read and paraphrase the mathematical expressions/operators one-on-one with a tutor for a basic comprehension. Furthermore we encouraged the trainees to use self-invented signs and symbols to comment on the mathematical expressions (Roam, 2008).

To further the exposure to 'symbol-shock' our team encouraged participants to

- Try to learn the alphabet of mathematical expressions on the internet ('mathematische Notation');
- Try to translate a mathematical expression into spoken natural language;
- Rewrite the mathematical term in blend of natural language together with mathematical symbols;
- Replace the most outstanding 'shock-symbols' with different and more intuitive symbols or words;
- Introduce symbols or pictograms with more intuitive meaning into the 'translated' formula
- Ask peers, teachers, parents etc. for help in understanding unfamiliar mathematical notations;
- Use intelligent computer software or smartphone and tablet apps; e.g. math-online on internet;
- Spend (more) time on mathematical tasks instead of avoiding and neglecting math-notations.

As a result of our explorative experiment our team was able to explain to the participants that science, especially mathematics and physics, may have much in common with a secret knowledge in occult science in antiquity. However today's scientific knowledge and scientific know-how is accessible to everybody and is not limited to only scientists in 'closed gate communities'. Then the team encouraged the participants to explore the many methods available for learning the basics of mathematics. These include illustrated textbooks, podcasts or open access internet-platforms such as Khan Academy (www.international.khanacademy.org).

According to Frege (Begriffsschrift, 1879) sometimes it is helpful to regard mathematics and its notation as a form of logic which is illustrated in a well-made graphic novel by Doxiadis et al. LOGICOMIX (2009).

Finally, as stated by Polya (2014), children and adults should simply be given the possibility to discover the meaning of mathematical problems by themselves and this should preferably occur in a form of a teacher-student dialog as learning is easier in joint efforts.

Conclusion

Symbol-shock – observed when people are faced with unfamiliar mathematical language and/or unknown mathematical symbols – may manifest itself in emotions such as fear, anger, and resignation etc. Symbol-shock is understood in this study as a combination or blend of cognitive and emotional states. To prevent the manifestation of symbol-shock we propose further exposure to mathematical formulas and symbols. In addition establishing a self-mental monitoring system to keep control of cognition and emotions can further benefit the young adults who lack the ability to comprehend the unfamiliar mathematical expressions and operators. By doing so, young adults may be able to give themselves the chance of not being overwhelmed by non-trivial mathematical notations. Therefore, let's talk about math!

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Resources

Mathematische Hintergründe. Variable, Terme, Formeln und Identitäten. mathe-online.
- http://www.mathe-online.at/mathint/var/i.html

Mathematische Notation.
- http://de.m.wikipedia.org/wiki/Mathematische_Notation#Mathematische Zeichen

Term. In mathematical logic a term denotes a mathematical fact (recursively) constructed from constant symbols, variables and function symbols.
- http://en.m.wikipedia.org/wiki/Term_(logic)

Begriffsschrift
- http://de.m.wikipedia.org/wiki/Begriffsschrift#Syntax and
- http://gallica.bnf.fr/ark:/12148/bpt6k65658c

Promoting Everyday Mathematics within the Framework of Employment Programs in the Canton Aargau, Switzerland

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Abstract

The pilot project 'Promoting Everyday Mathematics within the Framework of Employment Programs' (2008/2009) was the first of its kind in the Canton of Aargau, Switzerland. It was developed upon the request of the cantonal Office of Economics and Labor and targeted towards unemployed, often low-skilled persons with differing educational and professional backgrounds. Two different approaches were offered: one with a collective structure at the Stollenwerkstatt Aarau and one with an individual training structure at the Lernwerk Turgi. Didactically, both were designed to utilize the participants' prior knowledge and experience and to support them in the relevant questions and problems of their professional and personal daily life. This article focuses on presenting the individual modules of the course with the collective structure which took place at the Stollenwerkstatt Aarau: Its nine modules are described in detail and complemented with photographs. Furthermore, general insights from both approaches and the current status of the pilot project are presented.

Keywords: mathematics, framework, employment, adult literacy

Introduction

Surveys done on both national and international levels have indicated that apart from reading skills, everyday mathematics, the ability to solve problems and work in a team, and the necessary skills to deal with information and communication technology belong to the core competences required to cope with personal and professional daily life. By competence we mean the ability to respond successfully to complex demands within specific contexts. In the area of everyday mathematics, this means that one has the necessary knowledge and skills at his disposal 'in order to meet the demands of mathematical problems which present themselves in everyday life in an appropriate manner' (Notter, 2006, p.11, translation by the authors).

In spite of the fact that an international survey (Adult Literacy and Lifeskills [ALL], 2005), showed Switzerland to be leading in the area of numeracy competence, there were and still are some 400,000 people (8.6% of the population between 20 and 64 years) that have great difficulty in mastering simple mathematical problems in their professional and personal daily life. In addition to insufficiencies in everyday mathematics, the ALL Survey indicated that many of those affected are likely to have similar problems with the other basic skills such as reading and writing (cf. Kaiser, 2009, p. 4).

Various surveys conducted by AMOSA (Arbeitsmarktbeobachtungsstudien Ostschweiz, Aargau, Zug, & Zürich, 2007; 2008; 2013) in the eastern and central parts of Switzerland indicate that barring old

age and immigration origin, the primary cause of long-term unemployment for job-seekers is a lack of post-compulsory education. The longer the duration of their unemployment, the less chance they have to (re-)enter the labor market. (cf. AMOSA, 2007, p. 6; AMOSA, 2013, p. 26) Tables 1 and 2 below show these developments:

Table 1.
Educational level of the economically active population over 20 in the zone of AMOSA (AMOSA, 2013, p. 27)

	compulsory education		no information		housekeeping program
	professional apprenticeship		intermediate diploma school (DMS)		full-time vocational schools
	higher professional education		university and university of applied sciences		prevocational training
	European baccalaureate				
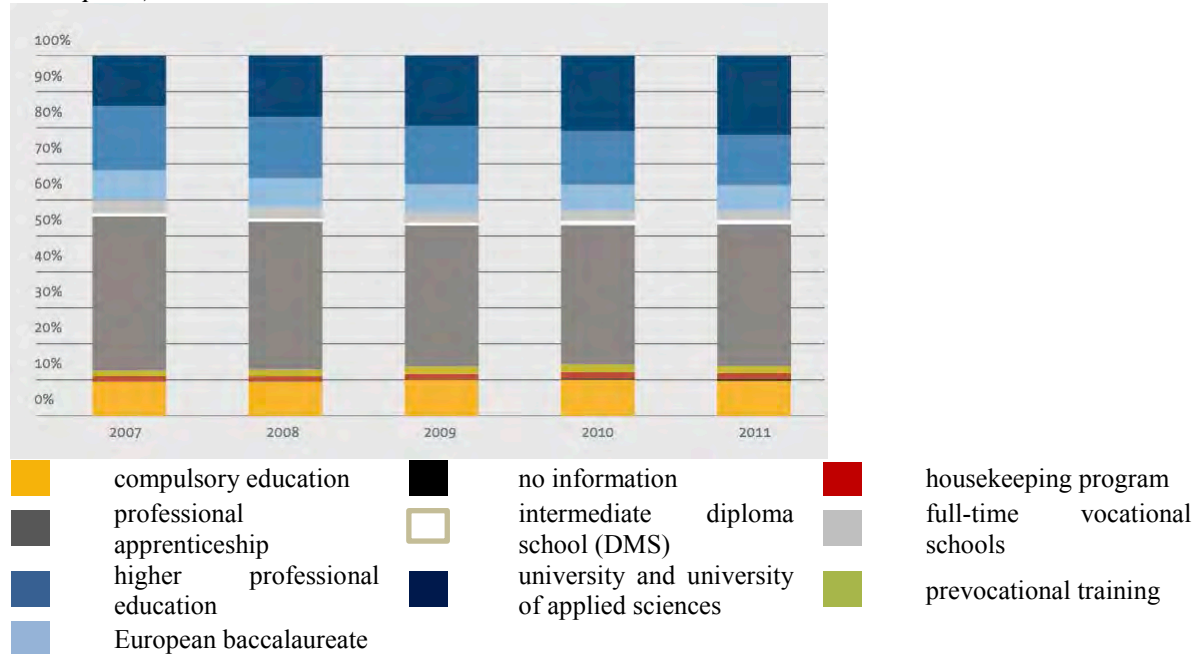
Table 2.
Development of the number and percentage of less-skilled job-seekers in eastern and central Switzerland (AMOSA 2013:27)
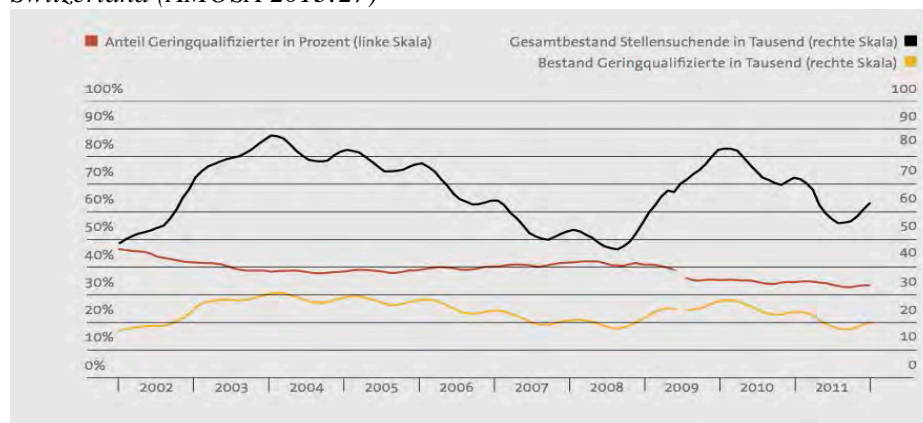
- Total of all job seekers in thousands (**black line,** right scale)
- Percentage of those who are low qualified (**red line**, left scale)
- Total of the low qualified in thousands (**yellow line,** right scale)

In recent years it has become obvious that people possessing low professional qualifications have great difficulty finding new employment – even when unemployment is on the decrease. In order to effectively and sustainably increase the employability of non-native speaking and/or low-qualified job-seekers, Canton Aargau participated in two pilot projects: one in the area of language development 'on the job' (2007/2008) and the second in cooperation with Canton Vaud in the area of everyday mathematics (2008/2009). Both pilot projects were developed at the request of the cantonal Office of Economics and Labor and put into practice in cooperation with two providers of labor market

measures, namely Lernwerk Turgi and Stollenwerkstatt Aarau. Ernst Maurer and Hansruedi Kaiser, the authors of the SECO (State Secretariat for Economic Affairs) framework concepts (Maurer, 2010; Kaiser, 2009), supported the project group didactically. The two authors of this article were part of the project group and developed and led the courses in the Stollenwerkstatt's program.

This article will first briefly present the context in which these courses were developed and then focus on the description of the nine modules as they were implemented at Stollenwerkstatt. It concludes by describing that which was experienced in Stollenwerk during the course, and complementing it with the experiences at Lernwerk Turgi. In conclusion, the continuing development and current situation in Canton Aargau are described.

Context

Canton Aargau is one of 26 Swiss cantons which are the member states of Switzerland. In the Swiss political system, cantons enjoy a high degree of autonomy and are responsible for such areas as public education, welfare and health care. They also retain the power of taxation. As far as unemployment is concerned, the cantons are also responsible for implementing the so-called 'labor market measures'. According to Article 1a (2) AVIG (Unemployment Insurance Law), labor market measures serve to combat impending or already-existing unemployment. The primary goal is a quick and preferably permanent re-entry into the labor market.

It is of prime importance that the affected people increase the ease at which they can be placed. This must be done by utilizing appropriate education, activities or other special measures. An example of the latter might be the introduction of an induction allowance for new employers.

Even those involved in occupational programs (in Stollenwerkstatt Aarau and Lernwerk Turgi) received internal consulting services and education. During 2008-2009 this accounted for almost 40% of their time in the program.

Table 3 illustrates the trend of unemployment in Canton Aargau: the inverse proportionality between the number of job seekers and the occupational places available. A further development for job seekers has been in the project The Gateway to the Labor Market Menziken in 2012, where the regional job centre works with the invalidity insurance and the social services of 10 political communities (inter-institutional cooperation) under one roof (cf. http://www.pforte-arbeitsmarkt.ch).

Table 3.
General framework in Canton Aargau (based on statistics from SECO 2009 and 2014)

Year	Number of regional job centre	Rate of unemployment	Number of places available
2008/2009	6	12,026 job-seekers 2.9% (CH 3.3%)	471 in 6 programs
2014	7 (including Pforte Arbeitsmarkt Menziken*)	13,224 job-seekers 2.7% (CH 2.9%)	323 in 7 programs

* The Gateway to the Labor Market Menziken

Collective Courses in the Stollenwerkstatt Aarau and Individual Coaching in Everyday Mathematics in the Lernwerk Turgi

Both organizations involved in the pilot projects are private providers of labor market measures as mentioned in the previous section. Stollenwerk Aarau, which was founded in 1991, merged with two partner organizations: the Storchenstrasse Association and the Pegasus Foundation. Together they formed a social corporation at the beginning of 2012, TRINAMO AG: 'TRINAMO AG makes long-term (re-) integration into the labor market and society possible – especially for the psychologically disadvantaged.' (See http://www.trinamo.ch).

At the moment there are 120 staff members in the various locations of TRINAMO AG in Canton Aargau. Within these 120 are course leaders, consultants, coaches, group and workshop leaders, IT and administration staff and employees from various entities (for example, restaurants). Currently, 85 places are available per year for job-seekers who receive unemployment insurance, another 85–95 places are for welfare recipients, 60 for invalidity insurance recipients and 20 places are reserved for those of reduced performance capability.

Lernwerk Turgi is an association and has been in existence since 1998. Its diverse programs and places of employment make it a place where job-seekers and the disadvantaged, particularly health-wise, can be advised, supported, engaged and equipped. (See cf. http://www.lernwerk.ch)

At the moment there are a hundred staff members at Lernwerk Turgi's various locations in Canton Aargau. They are involved in the following areas: integration projects, complementary courses and coaching, youth and educational projects, application for jobs workshops and the employment of those of reduced performance capability. Currently, 80 places are available per year for job-seekers who are recipients of unemployment insurance, 27 for welfare recipients, and 23 for invalidity insurance recipients. The places reserved for those of reduced performance capability (about 35) are included in the number of the staff members above.

Table 4 indicates the ways in which participants were involved in the pilot projects in Stollenwerkstatt and Lernwerk Turgi

Table 4.
Fields of work of participants in the pilot projects

Stollenwerkstatt Aarau and Wohlen	Lernwerk Turgi
carpenter's workshop facility management leather workshop glass recycling canteen	crea-atelier IT workshop kitchen bicycle workshop facility management laundry carpenter's workshop office workshop

The goal of the 3-6 month occupational program was to improve the employability of its participants by helping them to acquire practical work experience and receive job coaching and training. Through the furthering of core qualifications with relevant basic and professional expertise, the participants' chances of finding a job improve. For this pilot project the two providers offered the course within a collective and an individual training structure respectively: Stollenwerkstatt Aarau designed a modular course for its participants in their leather and recycling workshops. Lernwerk Turgi offered consulting services within the framework of job application coaching for individuals, which dealt with everyday mathematical questions. Each individual is assigned to a program as suitable as possible by a personal consultant who is employed at one of 6 regional job centres. During their attendance of an occupational program, job seekers can gather practical experience in job-related situations. They attend internal educational courses and are offered counseling and assistance by an individual job coach.

The pilot project aimed at adult job-seekers between the age of 21 and 55 with mainly low qualifications needed:

- to be labor market oriented
- to be oriented toward participants' practical needs
- to be solution-oriented and action-oriented
- to be holistic
- to promote confidence in one's own ability to learn
- to promote self-confidence by acknowledging existing learning strategies
- to increase skills and knowledge
- to be conducted on the basis of voluntary participation

The project was deliberately offered in two different formats. The Lernwerk Turgi developed individual coaching for job-seekers within the module Job Application. The Stollenwerkstatt Aarau designed a modular course for participants in its various workshops – for example, in the areas of glass recycling, leather work, carpentry, and canteen and facility management. Both pilot projects were aimed towards the same target groups, taking into account their resources and their practical mathematical needs. However, the educational programs did not contain traditional math training courses.

Didactics and Methodology

The didactics and methodology for the individual and collective courses in numeracy for adults, were designed along the lines of the concept of cognitive apprenticeship (Collins, Brown, & Newman, 1989; Kaiser, 2009). It was important for us that the group leaders assess the relevant practical everyday mathematical needs in the individual work groups. In this way the individual and collective educational modules were able to meet the needs in an appropriate manner. Table 5 shows the specific everyday mathematical requirements needed by the various workgroups in Stollenwerkstatt Aarau and Lernwerk Turgi (Kaiser, 2009). It is interesting to note that one organization identified these needs according to its fields of work, namely the Lernwerk Turgi (right column Table 5), while the other presented it in form of mathematical themes. Within the framework of individual coaching, the Lernwerk Turgi additionally assessed the previous knowledge of program participants. Each student constructed his or her own two-dimensional pyramid, specifically designed for self-evaluation (see Figure 5 for a pyramid example).

Table 5.
Needs assessment in everyday mathematics, Stollenwerktstatt Aarau and Lernwerk Turgi, 2008

Stollenwerkstatt Aarau and Wohlen	**Lernwerk Turgi**
Measuring & measures • knowing systems and units of measurement for length, weight, volumes, speed and money • being able to measure surfaces and simple three dimensional figures • being able to convert specifications from one unit into another	**Crea-Atelier** • calculating costs for material • knowing prices for square and running meters • being familiar with division (fractions)
Number line and arithmetic • being able to order and compare numbers • calculating with and without a calculator (column addition, etc.). • having a general feeling for numbers, both positive and negative, percentage, ratios and fractions	**Gastronomy** • changing money • calculating change • calculating costs for chips • being familiar with quantity units • being familiar with kilogram, gram, deciliter, liter • knowing and applying units, weighing • being familiar with multiplication (e.g. the simple rule of 3, cross-multiplication)
Approximating results • understanding units and dimensions of a given problem • having a feeling for adequate accuracy	**Facility management / laundry** • dispensing washing powder • knowing units such as ml, dl, etc. • being able to dilute proportionally, e.g. 1:5
Percentage and wages Plans and visual representations Proportions Probability	**Area / bicycle workshop** • being able to add, subtract and divide • reading plans • converting scales • doing mental arithmetic while putting together the bill

More specifically, the following approach was used for the group work. Along the lines of the concept of cognitive apprenticeship, participants in the Stollenwerkstatt Aarau received practical everyday mathematical tasks adapted to their own existing knowledge and skills. For this they worked in small groups of 2 – 4 people. After having worked in these groups, each team presented its approaches and solutions. Both the merits and difficulties of the various solutions were considered by the students and teachers. Afterwards possible alternative approaches were sought (Kaiser, 2009)

Experiences from the Stollenwerkstatt Aarau

At the Stollenwerkstatt Aarau a course consisting of nine modules was developed and implemented (November 2008 – March 2009). There were a total of 24 participants in this collective part of the pilot project. The group was very diverse in terms of age, first language and educational background, but a majority of the participants represented the target group of low qualified job seekers. As each module was designed to be self-contained, the participation varied for all of them. Five to twelve individuals attended each module. We noticed that a core group of five participants was significant for the success of the pilot group. Their interest and increasing zeal for everyday mathematics was contagious within and without the group. We were able to build on their experiences and to cater to their needs. They were also the ones who benefited the most and expressed their desire to see the program continued. The following list presents an overview of all modules and their thematic foci regarding mathematical content as well as relevant every day situations.

Module 1	**Introduction, recipe and production of caramel candies**
	(Percentage, Compad)
Module 2	**Wage statement**
	(Gross and net wages, deductions, social security, percentages)
Module 3	**Making a leather bag**
	(Square measures, right angle)
Module 4	**Apartment plans and rental contracts**
	(Ratio, square measures, gross and net rent)
Module 5	**Budget, costs of a car with and without leasing contracts**
	(Basic operations, measures of capacity, percentage, payment by installments, value added tax VAT)
Module 6	**Tax declaration**
	(Financial assets, statutory deductions)
Module 7	**Comparing supermarket prices (Aldi Suisse, Aldi D, Migros)**
	(Shopping basket, consumer behavior, travel expenses, cashier, giving change)
Module 8	**Sudoku, group interview**
	(Calculation, strategies, logic)
Module 9	**Visiting the glassmaking factory in Hergiswil**
	(Physics in daily life)

Each module was designed to take place within half a day and consisted of three units of 45 minutes each. The given time frame was flexible depending on how the module developed and on the needs of the participants. Based on the participants' initiative, the initially foreseen seven modules were complemented by two additional ones. These were used for both group interviews (Module 8 in the list above) and Sudoku. Our appreciation and thanks were expressed to the participants by inviting them on a field trip. The field trip lasted an entire day and was spent with a visit to a local glass factory on Lake Lucerne (Module 9).
In the following paragraphs each of the modules is described in more detail. Furthermore Figures 1-4 show some pictures with specific materials and scenes from these modules.

We carried out the first module in preparation for a Christmas sale and focused on the production of caramel candies and percentage calculation. In addition to percentage calculations, participants' estimation skills were also tested. The work was carried out in groups. The participants were allowed

to use Compad's[6] support materials to find their solutions (see Figure 1 above). These were sometimes used during the solution process to visualize the solutions of individual group members. The calculated results were shared with each other at the end, as described above. The approach was relaxed and productive. When it specifically came to convincing other groups of the correctness of the results and the corresponding solutions, a constructive competition arose between the groups.
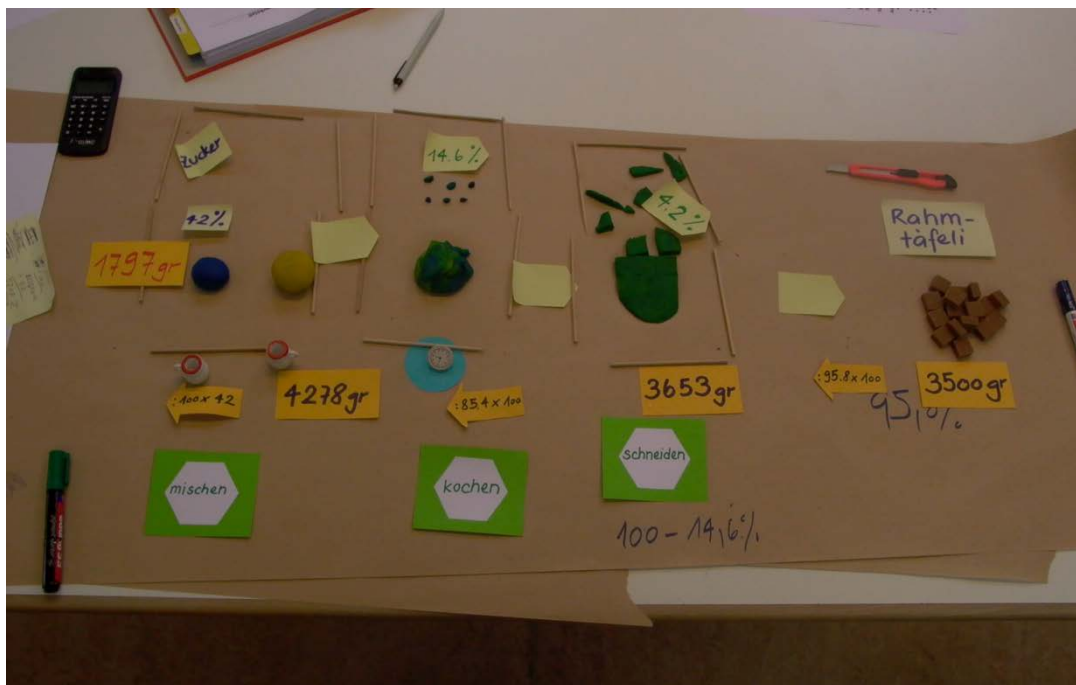
Figure 1. Module 1: Introduction, everyday mathematics using a recipe and production of caramel candies (percentage, caramel candies)

The second module was devoted to the subject area of pay statements. It dealt with gross/net salaries, social security deductions and took up percentage calculations from the previous module. There was an exciting debate and participants approached us with many specific questions. Most participants are constantly surrounded with thoughts about earnings and daily allowances due to their particular life circumstances. This module also helped them to gain a deeper understanding of the social security system in Switzerland. Again, the participants calculated in groups to identify the specific deductions made by employers or by the unemployment insurance in a pay statement. They then compared them with the net wage. When the working groups presented their results in plenum, they discussed the meaning and the percentage calculation of the various social insurances. During this module the atmosphere was quite emotional.

We would like to present the third module in more detail. It included the analysis of surface masses and right angles (see Figure 2 above). In line with the basic didactical approach, work was carried out in groups. Work in the leather workshop requires the drawing of right angles – a prerequisite for the production of bags. To avoid 'leftovers', the hides and animal skins had to be used as economically as possible. Therefore, we devoted ourselves to this area in the course unit. The task was again oriented towards real life conditions and included the use of relevant tools. We noticed that participants were often not familiar with tools such as protractors and they quickly reached their limits trying to use these tools. It was important for us to see that although some participants could solve the task mathematically, others also made use of resources such as rulers and triangles. To facilitate this, animal hides were modelled on paper. The participants thus had the opportunity to re-enact the cutting

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[6] Compad® (**com**munication **pad**) is a means of communication and is the name for a collection of tested, multi functional and flexible teaching and learning materials. It allows groups of people to present specific content, models or processes in a three dimensional, creative and visible manner. More information in German is available at: http://www.schulverlag.ch/page/content/index.asp?MenuID=3154&ID=5308&Menu=1&Item=20.7.5.1 [10.10.14]

required in bag production on fake hides. Besides the mathematics, handicraft skills were also required for the cutting task; a combination that made it possible for participants with inadequate mathematical skills to raise their profile and self-worth in other areas.

Figure 2. Module 3: Making a leather bag (square measures, right angle)

Module 4 dealt with raising participants' awareness when dealing with apartment advertisements and floor plans (see Figure 3 above). The objective was to give the participants the skills necessary to read floor plans and to recognize the relationship between apartment size and rent. In addition, a discussion took place about the 'hidden' costs, such as the incidental costs or the net and gross rent. Most of the participants were able to report on the subject from their own experience. This led to a positive learning atmosphere.

Figure 3. Module 4: Apartment plans and rental contracts (ratio, square measures, gross and net rent)

Vehicle costs with or without lease agreements, tax declarations and price comparisons between major Swiss distributors and those in the neighboring countries were other issues with we dealt with in our pilot project. It could be observed that the more a subject touched the personal life of the participants, the greater the intensity of the dialogue and their commitment to deal with the task.

One module also involved Sudoku because we believe that mathematics should be fun as well. A group interview was held to gain deeper insight into the previous practical experiences and attitudes of the participants towards mathematics as well as their assessment of this pilot project in Stollenwerkstatt Aarau. In methodology the group interviews focused on the documentary method and the analysis of group discussion (Nohl, 2009). We believe that further research on practical learning experiences with natural working and learning groups is needed and could be very insightful in the area of informal learning.

On the last course day, we visited the glass factory in Hergiswil on Lake Lucerne (see Figure 4 below). The exhibition there presented everyday physics using impressive glass objects. The joint field trip was a perfect opportunity to conclude this pilot project in a special setting with all of the various work and workshop groups. It was an ideal opportunity to express our appreciation and respect towards the participants for the joint learning process and for the many beneficial opportunities of teaching and learning which had taken place in the past months.

As course as leaders, we would like to add that it was a wonderful experience to see that learners who had had difficulties with mathematics in school, could renew their interest and motivation to deal with mathematical issues as adults and to gain self-confidence and expertise. Others could build on successful past experiences and build bridges to their present situation. For example, one of the female participants mentioned with a new self-confidence that in her family, it is she who is responsible for the finances.

Figure 4. Visiting the glazier's workshop in Hergiswil (physics in daily life)

Summary of the Experiences from Stollenwerkstatt Aarau

Overall, the following observations can be made from this collective part of the pilot project:
- The course was carried out by committed and motivated groups of learners.
- A core group of approximately five participants (and course leaders) carried the momentum.
- The close cooperation between the departments for work/production and education was essential.
- Team teaching was experienced as an asset.

- A relationship of trust between the course leaders and participants was essential for voluntary participation and for a positive learning experience.
- Teaching and learning outside of the classroom allowed a 'new approach' to teaching and learning mathematics; this was really based on everyday experiences and needs of the participants.
- Group work decreased inhibition and allowed synergies to occur.
- Presenting and discussing a variety of solutions with both their strengths and weaknesses, helped to focus on participants' resources and gain relevant knowledge and mathematical understanding.
- Understanding the mistakes of the participants and using them as an opportunity is an important element in learning.
- Participants can create their own learning aides.
- Individual calculation problems might go unrecognized.
- Individual practice and deepening is necessary.
- Group interviews allow for deeper insight into the motivation, the educational biography and learning experiences of the participants and can be used for further research (Nohl, 2009).

In the next section we will briefly present some of the experiences made with the part of the pilot project that used an individualized approach in order to complement the experiences mentioned above.

Individual Training Structure in Lernwerk Turgi

In Lernwerk Turgi, the training program for everyday mathematics was integrated into the educational module 'job application'. It was carried out by an individual coach.

In the beginning, each participant (mainly low-qualified job-seekers) needed to lay out a pyramid assessing his or her current knowledge and mathematical skills (see Figure 5 below). Jointly the coach and the participants evaluated the results. This procedure allowed specific, individual statements about target job application strategy to be formulated as well as the participant's learning needs in everyday mathematics. As a result, an individual proposal for filling these gaps in everyday mathematics was planned. The coach recognized the mathematical needs (demonstrated in Table 5 above) as well as the relevant subjects drawn in the pyramid (see Figure 5 below). On the basis of practical exercises, the themes were worked on: for example, the giving of correct change, or recognizing and utilizing measures used in the canteen or in an internal company laundry and facility management. The containers shown in Figure 5 were used to estimate and to compare different sizes and measures. The comprehension of the syllables 'deci', 'centi' and 'mili' were discussed and illustrated in everyday life.

The individual training structure in Lernwerk Turgi

The following are the characteristics of this individualized part of the pilot project.

- It focused on specific questions of the learners,
- It enabled rapid progression in learning,
- Group dynamics were missing, and
- No group resources were included.

The pilot project indicated that an individualized course structure would need a larger amount of resources (e. g. time, staff members and finances), in order to make it available to an increasing number of participants. The coach reported that working one-on-one was very intense and the absence of peers was a disadvantage. Ideas among equals could not be exchanged, and healthy competition and acknowledgement among the participants could not take place.

Figure 5. Assessing individual strengths and weaknesses using a pyramid in Lernwerk Trugi

Figure 6. Individual coaching of everyday math in Lernwerk Turgi: units such as millilitre, decilitre, litre, quantity units

Further Developments and the Current Situation in Canton Aargau

Today, everyday mathematics has been integrated into the internal training modules of the working groups' canteen/gastronomy and facility management at TRINAMO, as the Stollenwerkstatt Aarau is called today. The Lernwerk Turgi as well as the Foundation Wendepunkt, which is a further provider of labor market measures, both offer one training course in everyday mathematics which is organized

in a cross-cutting manner and aimed at all groups. The program in everyday mathematics as it is offered by the Foundation Wendepunkt is particularly targeted to reach people with an immigration background. The Lernwerk Turgi has continued to provide support in everyday mathematics since the inception of the project in 2008/2009 as part of its 'job application' training. Instead of the original individual coaching however, the current course leader is further developing the format of a collective training for job-seekers. The half-day training in everyday mathematics which is offered approximately once a month is an integral and mandatory part of the complete educational offer and is also oriented towards the needs of heterogeneous course groups. They usually consist of people between the years of 18 and 64. Whereas during the pilot project from 2008/2009 only individuals in the context of labor market measures were participating in the course (supported by the unemployment insurance), the current groups also include people who participate because they were assigned by the disability insurance or by social services.

We visited a module of an everyday mathematics course at the Lernwerk Turgi in May of 2014, in which 8 people participated. The half day module took place from 9:00 to 11:30 am in a bright, spacious room in with a large table in the middle as well as several computer work stations with access to the internet. Contrary to the pilot project this was a separate course room and the class did not take place in one of the workshops. The content of this module was the planning of a trip with the help of a map. The planned distances were supposed to take as little time as possible, and the use of petrol, as well as the other costs, were to be calculated as realistically as possible. Among others, the following competences were included: reading street maps, estimating and calculating distances and the use of the rule of three in practice. Tools included a whiteboard, a map of Europe, a calculator, notepaper, the available computers as well as the written task in simple German language.

As was already the case in the pilot project in both organizations we could observe that the course leader successfully managed to create an interest and motivate also those participants for active participation who thought that they had problems with and little interest in mathematics. In addition to the set task the participants contributed their own questions such as trying to calculate the flight distance from their home countries to Switzerland and compare costs of transport. The participants worked in groups and found the relevant information on the internet. Afterwards the various groups compared their different procedures as well as their insights – a continuation of the approach installed in the pilot project. Building on the insights from the pilot project the Lernwerk Turgi developed further modules and material in everyday mathematics which can be used flexibly as required by the current needs and interests of heterogeneous groups as well as by the individual knowledge. The current course leader who was already contributed actively to the pilot project considers it particularly important to keep everyday mathematics very close to practice and not to professionalize it too much or present it in a school like manner.

In order to further develop everyday mathematics for job seekers with a particular focus on the integration into the workforce, we think that an awareness for what is often 'invisible' everyday mathematics is needed – both on the side of employers and employees. In addition regular and targeted opportunities for the further education of the course and group leaders are key. Furthermore – in institutions like the Stollenwerkstatt Aargau (TRINAMO) and the Lernwerk Turgi – a close cooperation between the areas of work respectively the workshops and the training areas is key as is an official commitment to everyday mathematics in the educational concept of such providers of labor market measures. This requires additional personal and financial resources. The new law for further education, which was adopted by the Swiss parliament on June 20, 2014 (Bundesgesetz über die Weiterbildung)[7] has laid the foundation for such developments. In this sense, the Office of Economics and Labor of Canton Aargau considers a target group and labor market oriented training offer in everyday mathematics for all programs of temporary employment for its region.

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[7] The goals of the new Federal Law on Further Education stipulate that: (1) conditions be created which make further education possible for everyone, specifically for the under skilled. (2) chances for the under qualified to enter the labor force be increased.

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Simulating Outside-the-Classroom Maths with In-Class Word Problems

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Abstract

Not all word problems are designed to simulate real-life situations, but for those that are, a framework proposed by Dr. Torulf Palm can serve as a guide to developing authentic student tasks and assessments, at least as authentic as possible in the classroom. For adult students, given their life experiences and educational needs, that authenticity may be particularly important. The framework's benefits are explored.

Keywords: simulation, mathematics, real life

Introduction

Math students often ask, 'When am I ever going to use this in real life?' That is a valid question, as students are interested in learning what will be relevant and helpful to them. However, word problems are often contrived and not real-life. While not all word problems are designed to simulate real-life situations, it is possible to develop and evaluate word problems in terms of how closely they simulate an actual situation that students will encounter outside the classroom. This article will introduce a framework for doing just that. First, the idea of simulation will be discussed. This is followed by a framework proposed by Dr. Torulf Palm. Finally, benefits of the framework are noted as well as issues that are raised for practitioners.

Simulation

Flight Simulation Criteria

It is first necessary to talk a little about simulation. An interesting way to view simulation is by using an example of flight simulators used for training pilots. The International Civil Aviation Organization (ICAO) has a set of criteria for the qualification of flight simulation training devices (ICAO, 2009). Note how specific the criteria are for just the cockpit/flight deck layout and structure:

> *An enclosed, full scale replica of the cockpit/flight deck of the aeroplane being simulated including all: structure and panels; primary and secondary flight controls; engine and propeller controls, as applicable; equipment and systems with associated controls and observable indicators; circuit breakers; flight instruments; navigation, communications and similar use equipment; caution and warning systems and emergency equipment. The tactile feel, technique, effort, travel and direction required to manipulate the preceding, as applicable, should replicate those in the aeroplane. (p. II-App A-3)*

For sound cues the criterion is that the simulator must have 'significant sounds perceptible to the flight crew during flight operations to support the approved use' and with 'comparable engine, airframe and environmental sounds' (ICAO, 2009, p. II-App A-19).

The atmosphere and weather section of the environment simulation includes a 'fully integrated dynamic environment simulation including a representative atmosphere with weather effects to support the approved use' (ICAO, 2009, p. II-App A-45) and also:

> *The environment should be synchronised with appropriate aeroplane and simulation features to provide integrity. Environment simulation should include thunderstorms, windshear, turbulence, microbursts and appropriate types of precipitation. (ICAO, 2009, p. II-App A-45)*

As we would hope, the criteria are very specific and we would want our pilots to have that level of simulation during training. This is especially true for simulating emergencies which would be difficult to train for while actually flying.

What level of simulation is necessary for a mathematics situation? The framework below contains a specific list of criteria. The degree to which they are followed depends on the extent that we want word problems to look like outside-the-classroom problems.

Mathematics Simulation Criteria

Palm (2006) proposes a framework that can be used when developing realistic word problems. It can also be used to evaluate the extent to which existing word problems are real-life. Table 1 is adapted from Palm's work and contains all of the criteria in a summarized list. The reader is referred to Palm's article for complete definitions and explanations.

Table 1.
Framework for word problems as simulations of real-world situations.

Criteria (aspects)		Defined as the degree to which:
Event	Situation	a student has a reasonable chance of encountering the described task outside of the classroom
	Question	the question in the school task is concordant with the actual event
Information/ Data	Availability	the information available in the task matches the information available in the actual event
	Realism	the information in the described task is realistic relative to the actual event outside of the classroom
	Specificity	the specificity of information in the simulated task matches that of the out-of-classroom event
Presentation	Mode	the method of information delivery in the task is similar to the actual event (for example, oral vs. written or through diagrams or tables)
	Language use	the language used does not detract from students using the same mathematics that they would in the actual event
Solution strategies	Availability	relevant solutions strategies, available to students, match those available outside of the classroom
	Experienced plausibility	the strategies experienced are plausible for solving the school task, as compared with the actual event
Circumstances	Availability of external tools	the tools available for the actual event are available in the classroom task (e.g., calculator, software)
	Guidance	explicit or implicit hints are available in the classroom task as compared with cues in the actual event
	Consultation & collaboration	there is a match, between the classroom task and the actual event, in the amount of help that is available from others
	Discussion opportunities	the student can ask questions and discuss the task in a manner similar to an actual event
	Time	time restrictions on solving are consistent between the classroom task and the actual event
	Consequences	there is similarity in the pressures and motivations of solving the task/event, based on success or failure
Solution requirements		judgments on the validity of solutions/methods are in concordance with the actual event
Purpose	Figurative context	the purpose of completing the task is in concordance with the event
	Social context	the purpose within the social context of the school task matches the purpose of the out-of-school event

Note: Adapted from Palm, T. (2006). Word problems as simulations of real-world situations: A proposed framework. *For the Learning of Mathematics, 26*(1), pp. 42-47.

As can be seen in the table, the framework is quite comprehensive with 18 criteria in seven different areas. The framework, or criteria, can be useful in two ways: designing word problems and evaluating existing problems.

Designing word problems

Here is how the criteria could be used in the instructional design process. Applicable criteria from the framework are noted in brackets:

Analysis:

- What is the learning outcome?
- Is a real-world simulation appropriate and to what degree?
- What are the relevant solution strategies available to the students and do they match the expected learning outcome? [Solution strategies – availability and plausibility]

Problem Design:

- Pick a situation and a corresponding question that a student has a reasonable chance of encountering outside of the classroom. [Event – situation and question]
- Decide what information will be available to the student and the degree of concordance with the real-world situation. [Information/Data – availability, realism, and specificity]
- Describe the problem context in a way that matches the degree of concordance desired and is appropriate for the student's level of language proficiency. [Presentation – mode and language use] State or indicate a clear purpose for completing the task. [Purpose – figurative and social context]
- Provide any hints or guidance that may be appropriate, given #3 and #5. [Circumstances – guidance]
- Design any consequences, if desired, that would match the real-world situation. [Circumstances – consequences]

Implementation Design:

- Decide on the tools that will be available to the student. [Circumstances – external tools]
- Determine the appropriate amount of time the student will have to solve the problem. [Circumstances – time]
- Determine the extent of any degree of collaboration or discussion that will be allowed during the problem solving time. [Circumstances – consultation, collaboration, and discussion]

Evaluation Design:

- Considering the analysis questions above, decide how the student's solution will be assessed. [Solution requirements]

Evaluating word problems

Palm and Burman (2004) used certain aspects of the framework to compare the realism of national assessments in mathematics for two countries. Their stated purpose 'was to describe in what way and to what extent the 'applied' tasks in the assessments are in concordance with task situations in real life beyond school' (p. 5). Not all of the 18 criteria were used because some were not that useful or applicable for a large-scale assessment (for example, the criteria of *consultation and collaboration*).

Primarily, two classification categories were used by the researchers. These categories 'correspond to whether the tasks were judged as simulating the specific category to a reasonable degree (Category 1) or not (Category 2)' (p. 8). Three categories were used for *specificity of information/data* where a judgment could be made that the word problem partly simulates real-life.

I believe that three categories are possible in many of the criteria. I have taken a few of what I consider to be the primary criteria and developed a rubric for which evaluation may be based on three categories of concordance. The rubric is shown in Table 2. The rubric has high, medium, and low concordance between a mathematical word problem and the out-of-classroom experience it is designed to simulate. As an example, for the *event* criterion, low concordance refers to little or no chance that

the student will experience this situation or have the same mathematical question. The *medium* classification allows for some chance that the student may one day be in that situation. The *high* classification allows for a 75% or higher probability.

Table 2.
Rubric for analyzing word problems as simulating real-world mathematical situations

| Criteria | Quality of Concordance[1] | | |
	High	*Medium*	*Low*
Event	There is a 75% chance or higher that the student will see this situation and have the same question. ☐	There is some chance, between 75% and 25%, that the student will see this situation, with the same question. ☐	There is little to no chance that the student will experience this situation or have the same question. ☐
Purpose	The purpose of the task in the word problem is explicitly clear and matches the actual situation. ☐	The purpose is somewhat clear and reasonably matches the actual situation. ☐	The purpose is vague and unclear. ☐
Information availability	The relevant information is the same as is accessible in the actual event, including whether or not it is explicitly stated or has to be retrieved. ☐	The information is similar to what is found in the actual situation but is somewhat simplified or is missing some normally available data. ☐	The relevant information in the word problem is clearly not the same as would be available in the actual event. ☐
Information realism	The context of the word problem is specific and the data are realistic according to what one might actually see. ☐	The context is not specific but the subjects/objects of the mathematical treatment are specific. The data is reasonably realistic. ☐	The context and subjects/objects are not specific and the data is not realistic. ☐
Guidance	The same guidance is available that would exist in the event. ☐	The word problem contains one minor cue or hint. ☐	The word problem contains a major cue or multiple hints. ☐
External tools	Students are able to use the same tools (calculator, map, computer, etc.), if any, as in the actual situation. ☐	Students are not able to use any tools but are given outputs of those tools. ☐	There is a complete mismatch in the tools available. ☐

The degree of concordance between a mathematical word problem and the out-of-classroom experience it is designed to simulate.

Benefits

A benefit of the above framework, as shown by Palm and Burman (2004), is to conduct research on concordance with real life. They stated: 'The tools of analysis, partly used in this paper, may function as an instrument for validation of the links between school tasks and real life task situations beyond

school' (p. 30). Also, this operationalizing of real-life concordance provides other possibilities such as checking the correlation of student performance on assessment tasks with levels of concordance.

An additional benefit is to help curriculum designers, textbook authors, and practitioners think more deeply about 'application' problems. All too often there is an assumption that contrived word problems can help students in solving real-life problems. If there is a concern about students actually being able to solve certain types of real-life problems, then we want to simulate those problems as closely as possible.

Issues

I will conclude with a few thoughts about several issues that must be considered with simulating word problems. These are described briefly below.

What is realistic?

If the goal is to have realistic problems, it may be interesting to consider a larger view of realism. Van den Heuvel-Panhuizen (2005) notes that even a fairy tale word problem may be considered realistic:

> However, it must be acknowledged that the name *Realistic Mathematics Education* is somewhat confusing in this respect. This all has to do with the Dutch verb *zich REALISE-ren* that means *to imagine*. This implies that it is not authenticity as such, but the emphasis on making something real in your mind that gave RME its name. For the problems presented to the students, this means that the context can be one from the real world, but this is not always necessary. The fantasy world of fairy tales and even the formal world of mathematics can provide suitable contexts for a problem, as long as they are real in the students' minds and they can experience them as real for themselves. (p. 2)

Are all maths relevant for simulation?

It seems clear that basic arithmetic and pre-algebra can be made very relevant in terms of simulating potential events that students may encounter. But, the intermediate algebra level is perhaps not so easy to find real-world applications that are ripe for simulation. Palm and Burman (2004) found that the tasks in the lower courses in Swedish and Finnish school national assessments were more in concordance with actual events than the higher courses:

> The high proportion of applied tasks in the tests included in this study could serve several of the purposes outlined in the introduction, such as providing good opportunities for students to experience strong links between school mathematics and real life beyond school. This is especially true for the tests used in the lower courses, which display a higher proportion of applied tasks than do the tests for the higher courses. (p. 31)

It is not desirable or necessary for all word problems, at any level, to be based on an actual event and simulated in the classroom. This leads to the next point about intent.

What is the intent?

It is very important that curriculum designers and teachers articulate the intent of tasks that students are expected to do. One of the decisions that has to be made is to decide if using a word problem in the classroom is (a) to add some relevance and interest or (b) to simulate an event that students have a good chance of encountering outside the classroom. If it is the latter, then the framework and criteria can serve as a help to developing high concordance with the actual event.

One interesting way to think about intent is to consider the concepts of horizontal and vertical mathematization. Van den Heuvel-Panhuizen (2010) interprets the earlier work of Treffers and describes vertical mathematization as 'moving within the world of mathematics' while horizontal is 'going from the world of real-life into the world of mathematics' (p. 4). I believe that some curriculum designers and teachers are ambiguous about this intent.

Examples of not considering intent and/or not communicating intent accurately are particularly found in intermediate algebra textbooks for developmental/remedial, tertiary prep students in the United States. The introductory information of one textbook has an applications index intended to show the

relevance of this edition: 'Real-world and real-data applications have been thoroughly updated and many new applications are included' (Martin-Gay, 2013, p. xvii). The index has 24 categories and over 450 sub-categories of real-world applications. Sounds great. However, the promise of real-world problems turns into contrived word problems that are more concerned with vertical mathematization. I looked at the list and picked out a problem type that sounded like it had a good chance of being real-world. In the category 'home improvement' I picked the sub-category of 'dimensions of a room.' I was directed to page 484 which had the following problem: 'The area of a square room is 225 square feet. Find the dimensions of the room.' I cannot think of one possible reason why anyone would know the area but not the dimensions of a square room. My fear is that when we say a problem is real-life and it clearly is not, this will breed cynicism in students. This is especially important for adults who have some life experience and can clearly see when a word problem is contrived. As stated by Palm and Burman (2004):

> If the irrelevance of the question is obvious to the students this may instead facilitate the forming of the belief some students have (e.g. Palm, 2002) that solving applied tasks is a game, with rules not necessarily consistent with the rules of real life problem solving. (p. 32)

Transfer of learning

Finally, a concluding thought on transfer of learning. I would not fly in a plane if it was the pilot's first airborne flight after only training on a flight simulator. Even if the simulator was certified by ICAO to meet all of the necessary qualifications, the prospect that the pilot could successfully fly an actual airplane would be in doubt. If our goal is to simulate an event that students may experience in the future, we need to realize the limitations of the classroom in simulating situations, regardless of having high concordance between a mathematical task and the actual event. As stated by Palm (2006):

> It is not possible to simulate all aspects involved in a situation in real life and consequently it is not possible to simulate out-of-school situations in such a way that the conditions for the solving of the task will be exactly the same in the school situation.

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What do I teach? Mathematics, Numeracy, or Maths?

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Abstract

There has always been a tension for adult numeracy teachers between teaching basic arithmetic (plus something about recognising shapes and reading charts) and satisfying learners' curiosity about various aspects of mathematics and number curiosities. In England, the publication of the Adult Numeracy Core Curriculum in 2001 defined the levels and content of adult numeracy for many. This is a live issue and one that can produce major conflicts over the policy and practice of teaching adults numeracy. Or is it mathematics? Or is it maths? These debates may begin with asking 'What is numeracy'? However, the discussion quickly requires us to ask 'What is mathematics?'

Keywords: math, numeracy, teaching, adult education, further education

Introduction

This paper is part of a personal project that began in 2002 at the ALM9 conference with a poster presentation. The project continues today and is partly an awareness-raising campaign and partly a political critique. I introduced the discussion in the ALM9 conference proceedings by stating:

> I work as a numeracy lecturer in Further Education in the UK, but I am very often described as the 'maths teacher', as that is a far more familiar term – especially to the students. Yet, it has become important to some of us who teach this subject to use the term 'numeracy' to describe the content of our teaching. (Kaye, 2003a, p. 102)

It was the paradox between the everyday comments from students and other staff, 'the maths teacher' label, and the formal description of the qualifications, policy documents and my teaching role that started me considering the significance of this nomenclature.

There was such a wealth of material available in the ALM proceedings that it seemed initially that I simply needed to catalogue what I considered were the most significant statements about numeracy in the proceedings; which was what I did in 2009. By the following year I realised I had to make clear my own objectives and how I evaluated the chosen quotations. I introduced my paper in the proceedings for ALM10 with the following aims:

1. To explore what is significant when numeracy is used, and when it is not used, and the word mathematics is used instead.

2. To explore what concepts are implied when we use numeracy and when we use mathematics.

3. To explore the meaning of numeracy – to define it, re-define it and share definitions. (Kaye, 2003b, p. 194)

I concluded the paper with the following declaration.

> I endorse the observations that numeracy is fluid and dynamic. In fact I now wish to take this further. I think the ideas and proposals that emerge from the discussions and debates

around the word 'numeracy' are the basis of the power of the concept of 'numeracy'. The ambiguity of the current meaning of numeracy enables the complex of ideas and concepts it represents to be continually redefined and extended. It enables questioning and inclusion, rather than enforcing acceptance and exclusion. (Kaye, 2003b, p. 198)

In many ways this is still what I am doing. I have presented these views at conferences, they formed the basis of a chapter in a book on teaching adult numeracy (Griffiths & Stone, 2013), and they were the topic of one of ALM's Webinars in 2014.

Through this period I have developed a greater focus on defining (or describing) mathematics as well as numeracy. More recently I have also been greatly troubled by the growing use of 'maths', in situations that seem inappropriate for an abbreviation. Some criticisms have been raised that this is an English-centric discussion, as many languages do not have an equivalent word for 'numeracy'. This is obviously a fact. However, I believe the debate about what mathematics should be, or is, being taught in post-compulsory education is taking place in less obvious ways.

In this paper I am not going to give an over-all analysis of the debate, but to continue awareness-raising by giving an annotated account based on the quotations used in the ALM webinar discussion and the conference presentation. Compared to previous papers there is now a greater focus on what has been said about mathematics, and I begin to open up the debate about using 'maths'.

What is Mathematics?

From a report commissioned by the UK government

There are positive senses in which mathematics is special. First, by virtue of its fundamental nature as a universal abstract language and its underpinning of the sciences, technology and engineering, mathematics has a claim to an inherently different status from most other disciplines Secondly, as we have set out above, mathematics is fundamentally important in an all-pervasive way, both for the workplace and for the individual citizen. (Smith, A. 2004 p 14, paragraph 1.16)

This gives a very clear statement that claims mathematics has a 'different status' to other subjects. This is a very significant claim considering this is in the context of a report advising on education policy. It also identifies the importance of mathematics in people's lives, whoever they are.

A view from the 13th Century

He who knows not mathematics cannot know the other sciences nor the things of this world . . . And, what is worse, those who have no knowledge of mathematics do not perceive their own ignorance, and do not look for a cure. Conversely a knowledge of this science prepares the mind and raises it up to a well authenticated knowledge of all things. (Bacon,1266, Opus Maius quoted in Fauvel J. et al., 2000, p.2)

Perhaps this quotation does not exactly identify key features within mathematics, but shows that the discussion of mathematics in society has certainly more than a 750 year history. It is also interesting to note that even here the importance of mathematics contributing to knowledge of other matters is noted.

From a Swedish study into learning mathematics

Mathematics is to be found everywhere, but to the individual it appears to be almost nowhere, a situation usually referred to as the relevance paradox of mathematics. An adult who feels anxiety and suffers learning blockages when faced with this subject is therefore likely to conclude that the subject is meaningless; it neither improves understanding of the environment nor adds to practical knowledge (Gustafsson. & Mouwitz, 2005, p.44)

These authors are active in the humanist tradition of education (referred to as 'bildung' in Swedish) and this comment is made in the context of them completing a research study for the Swedish government into mathematics education for adults. The article examines the differences between what they refer to as 'school mathematics' and 'mathematics proficiency for everyone'.

A comment on the 'traditional view' of mathematics

> Despite the time elapsed and the changes occurring, both in the educational world and outside, since [1976], many current commentators still appear to share a traditional view of 'mathematical ability'. It is seen as involving a set of abstract cognitive 'skills', which can be applied to perform a range of tasks, in a variety of practical contexts. This is considered to take place through a relatively straight-forward process of transfer. (Evans, 2000, p.2)

This could well be a useful example of what many people mean when they speak of 'maths', rather than 'mathematics. However, in this piece Jeff Evans is in fact more concerned with the ideas of transfer and how the mathematical techniques are learnt. It does provide a neat summary of the 'traditional view' though that is not something the author is proposing.

A commentary on the foundations of mathematics

> Mathematical statements are compelling, but their force is of a special kind; they are true, but their truth is uniquely defined. Mathematical reasoning is rigorous and deductive and mathematical propositions are simply the consequences of applying this reasoning to certain primitive axioms. Yet this in-grown, self-contained, iron-disciplined method is unlimited in its creativeness, unbound in its freedom. Neither arithmetic, algebra and analysis on the one hand, nor geometry on the other, are empirical sciences. Mathematics cannot be validated by physical facts, nor its authority impugned or subverted by them. Yet there is a vital connection between the propositions of mathematics and the facts of the physical world . . . Counting and measuring in the everyday world invariably parallel mathematical propositions but it is essential to distinguish between mathematical propositions and the results of counting and measuring. (Newman, 1956 p.1588 in re-issue)

This is taken from a publication that contains 133 extracts about mathematics selected by this author, James R Newman. It is called 'The World of Mathematics' and subtitled 'A Small Library of the Literature of Mathematics from A'h-osé the Scribe to Albert Einstein'. This statement summarises for me the view of an academic mathematician. The statement was made as part of the commentary on a section entitled 'Mathematical Truth and the Structure of Mathematics'.

A professional mathematician speaks

> Mathematicians can't bear to admit that there might not be an explanation for the way Nature has picked the primes. If there were no structure to mathematics, no beautiful simplicity, it would not be worth studying . . . Despite their randomness, prime numbers – more than any other part of our mathematical heritage – have a timeless, universal character. Prime numbers would be there regardless of whether we had evolved sufficiently to recognise them. As the Cambridge mathematician G. H. Hardy said in his famous book A Mathematician's Apology, '317 is a prime not because we think so, or because our minds are shaped in one way or another, but because it is so, because mathematical reality is built that way'. (du Sautoy, 2003, pp. 6-7)

This author is currently (2014) a professor of mathematics and the Charles Simonyi Professor for the Public Understanding of Science. This quotation, with a specific focus on prime numbers, serves to represent the view many mathematicians express of the 'beautiful simplicity' of mathematics. It is a view that speaks of a mathematical reality and of the significance of the structure of mathematics itself.

The philosophy of abstract mathematics

> When we think of mathematics, we have in our mind a science devoted to the exploration of number, quantity, geometry, and in modern times also including investigation into yet more abstract concepts of order, and into analogous types of purely logical relations. The point of mathematics is that in it we have always got rid of the particular instance, and even of any particular sorts of entities. So that for example, no mathematical truths apply merely to fish, or merely to stones, or merely to colours. So long as you are dealing with pure mathematics, you are in the realm of complete and absolute abstraction. (Whitehead, 1925, p. 394)

This is an important statement that represents many that have been made over the last few hundred years; mathematics is more about the abstract concepts, even if it begins in the material world, it is pure mathematics. For example, later in the same article Whitehead says 'Mathematics is thought moving in the sphere of complete abstraction from any particular instance of what it was talking about'.

In these few quotations I have tried to represent a range of views about what mathematics is. I am not arguing that any one of these is more important or more correct than any other. In fact, the most important point it that there exists considerable differences between what experienced mathematicians, philosophers, educationalists and teachers think is most significant about mathematics. In contrast to these statements I will now move on to look at what has been said about numeracy.

What is Numeracy?

The selection of statements about adult numeracy from papers published in the proceedings of the Adults Learning Mathematics international conferences formed the origins of this investigation. These were written by researchers and practitioners familiar with the everyday practice of teaching adult numeracy. A few of these are presented here, together with a selection of quotations drawn from official documents and policy statements.

The Adult Numeracy Core Curriculum

> 'Mathematics equips pupils with a uniquely powerful set of tools to understand and change the world' (The National Curriculum, (QCA). Changing the world may not be the immediate goal of adult learners, but being numerate – acquainted with the basic principles of mathematics is essential to functioning independently within the world. (Basic Skills Agency, 2001)

This quotation comes from the original paper version of the Adult Numeracy Core Curriculum. It is a very useful starting point for two reasons. It begins its definition by quoting a statement about mathematics (from the then Schools National Curriculum of 2000) as a definition of numeracy. It then makes a connection between 'being numerate', 'basic principles of mathematics' and everyday functionality. This is an exceptionally broad definition given the content of the curriculum document is entirely topic and technique based without context.

'The Cockcroft Report' on school mathematics education

> We would wish 'numerate' to imply the possession of two attributes. The first of these is an 'at-homeness' with numbers and an ability to make use of mathematical skills which enable an individual to cope with the practical mathematical demands of his everyday life. The second is ability to have some appreciation and understanding of information which is presented in mathematical terms, for instance in graphs, charts or tables or by reference to percentage increase or decrease. (DES/WO, 1982, p. 11)

The title of this significant report is 'Mathematics Counts' and it is described as the 'Report of the Committee of Inquiry into Teaching Mathematics in Schools'. It was chaired by Dr. .W. H. Cockcroft

and is generally referred to as the Cockcroft Report. The report refers to receiving many written submissions using the terms 'numeracy' and 'numerate' (paragraph 35). It also refers to the Crowther Report published in 1959 noting the word 'numeracy' was introduced then and that the meaning was 'intended to imply a quite sophisticated level of mathematical understanding' (DES/WO, 1982 paragraph, 36).

The view of an Australian researcher and practitioner

> One of the significant things in Australia which appears different from much practice overseas is that we are now actively using the word 'numeracy'. There seems to be almost Australia wide agreement that yes, we can use that word to talk about what we do – it isn't downgrading what we do, it isn't inferior to mathematics – and as we said in the introduction to the Adult Numeracy Teaching course: 'numeracy is not less than mathematics, but more.' . . . We believe that numeracy is about making meaning in mathematics and being critical about maths. This view of numeracy is very different from numeracy being just about numbers, and it is a big step forward from numeracy or everyday maths that meant doing some functional maths. (Tout, 1997, p. 13)

Dave Tout has contributed considerably to the debate about numeracy and here makes a case for numeracy being more than mathematics. Do you think numeracy is part of mathematics or is mathematics a part of numeracy? Or do they run in parallel over the same content? What is the dynamic between the two?

The concept of a 'numeracy incident'

According to Hoogland (2008),

> *In our study, we used as a starting point the definition of numeracy that is most widespread in the Netherlands . . . Which can be translated as: Numeracy is the combination of knowledge, skills and personal qualities that an individual needs to adequately and autonomously deal with the quantitative aspect of the world around us. From this definition we derived the concept of a 'numeracy incident'. The quantitative aspect of the world around us takes many forms. It shows up in artefacts and devices (meters, gauges, clocks, numbers, symbols), in constructions (measurements, angles, spatial attributes) and in texts (numbers, symbols, diagrams, maps, graphs, formulas) . . . A numeracy incident is thus a situation plus an individual's numerate response to the situation.* (pp 167-168)

This is an example of developing the concept of numeracy with a very strong emphasis on context. Not only a practical context, like we might find in measuring, but including how any one individual reacts and behaves in this situation. This provides an active or dynamic dimension to numeracy.

Numeracy for a meaningful life

> *Numeracy consists of being able to make an appropriate response to a wide range of personal, institutional or societal needs. To participate fully in everyday living, adults need the ability to understand broader contexts in which numerical demands are located . . . Here the knowledge of numeracy is seen as important, not just for utilitarian or abstract purposes, but as part of students' attempts to understand their own individual and collective lives and to make their lives meaningful.* (Benn, 1997, p.80)

Here the relevance to people's lives is extended even further. Here numeracy is seen as a tool for empowerment as an active citizen; numeracy is seen as having an overt political purpose.

Numeracy as an Analytical Tool in Mathematics Education and Research

Our two-pronged general definition of numeracy describes a math-containing everyday competence that everyone, in principle, needs in any given society at any given time:

- Numeracy consists of functional mathematical skills and understanding that in principle all people need to have.

- Numeracy changes in time and space along with social change and technological development. (Lindenskov & Wedege, 2001)

This is taken from a Danish publication which argues for greater use of the concept of numeracy in Denmark and the same authors have elsewhere devised a Danish word for numeracy *numeralitet*. They speak of their definition as an operational concept for a research project 'People's mathematical knowledge in technologies undergoing change'.

Numeracy as a process

According to Coben (2002), 'Numeracy is a notoriously slippery concept. There is no shortage of definitions but there is, crucially, a shortage of consensus, with the term meaning different things in different educational and political contexts and in different surveys of need (p.26). Also, Withnall (1994) posit that:

> At present, there is a tendency for policy-makers, practitioners and researchers alike to talk about 'numeracy' as though they shared a common understanding of its meaning. In examining some of the attempts to grapple with a definition already made, it has to be concluded that we do not, as yet, have an all-embracing operational definition with which to work. Nor, it seems is it necessarily desirable to do so. Numeracy must remain a fluid term capable of re-conceptualisation according to the contexts in which it is used and by whom. (p. 16)

These two quotations present another aspect of defining numeracy; the process of defining the concept has significance in itself. Diana Coben is a leading researcher into adult numeracy and in the article this is from proceeds to make connections between how numeracy is interpreted and what is relevant to teach. The second quote comes from the first Adults Learning Mathematics conference and I see this paper as part of the continuing the process of the 're-conceptualisation' of numeracy proposed by Alexandra Withnall.

What about 'maths' then?

I now want to give some consideration to the use of the term 'maths'. This may be an issue (if it is an issue at all) only for the UK (or may be England). I am pursuing it as I think it has greater significance. I am now asking 'What is maths?' Or perhaps more precisely: 'What do you mean when you say or write 'maths' rather than 'mathematics' or 'numeracy'?' Initially when I raise this the reaction from most people is simply to say, 'It is just an abbreviation', and therefore there is no reason to question its use at all. However, I could no longer accept this position when I saw it used in a government policy document.

In New Challenges, New Chances – Further Education and Skills System Reform Plan: Building a World Class Skills System published in 2011 in the UK by the Department for Innovation and Skills (BIS) 'maths' is used throughout. In the section 'Skills for Life: English and Maths for adults' the argument for a change of policy concludes with this evidence.

> There has been a large improvement since 2003 in Level 2 and above literacy, but no improvement in lower level literacy and the nation's numeracy skills have shown a small decline. So despite considerable efforts over the last 10 years to improve the basic skills of adults, our new national survey [*Skills for Life survey: headline findings* (2011), BIS research paper 57] shows that 24% of adults (8.1 million people) lack functional numeracy skills and 15% (5.1 million people) lack functional literacy skills. This is unacceptable. (BIS, 2011, p.10)

This is followed by a panel of 13 action points – here are five of them

- Re-establish the terms 'English' and 'Maths' for adults.

- Prioritise young adults who lack English and Maths skills, and those adults not in employment.

- Fund GCSE English and Maths qualifications from September 2012.

- Include the training of English and Maths teachers in the development fund for the sector to explore new models of delivering Initial Teacher Education.

- Ofsted proposes to increase its focus on the quality of teaching, learning and assessment in inspection. Paying particular attention to how well teaching develops English and Maths skills. (BIS, 2011, p.11)

This is a major policy paper by the UK government that quite specifically argues against using the term 'numeracy' and criticises the previous government's programmes for improving adult numeracy. (It is interesting to note that the term numeracy had to be used when referring to the skills surveys as they are based on measures of 'numeracy skills'.). However, it does quite specifically propose 'maths' should be used in the future. Is this really just using an abbreviation? I do not think so. Here is a statement about a new qualification currently (2014) being developed:

> *As a result of these concerns about the achievement of maths GCSE among learners in England, a number of policy changes have been initiated. The Department for Education (DfE) and the Department for Business, Innovation and Skills (BIS) plan to expand maths teaching in the FE sector, where most vocational subjects are taught, so that from September 2014 onwards:*
>
> *• GCSE maths or 'stepping stone' qualifications towards it will be taught to all students up to the age of 19 who do not hold this qualification at grade C or above; and*
>
> *• Level 3 core maths will be taught to the 22% of students who have achieved maths GCSE before embarking on a level 3 vocational course. (ETF, 2014)*

This new qualification is called 'Core Maths' in the documents produced by the Department for Education. See for example 'Core Maths Qualifications – Technical Guidance, published in July 2014. Interestingly in this document there is reference to GCSE, A and AS level 'mathematics', whereas the previous documents from BIS and ETF refer to 'GCSE maths'. I am not sure what the political or educational significance of this is yet. However, I think it is important enough to make a wider audience aware of this development, as we consider the implications of using the terms numeracy and mathematics.

What do I Teach?

What mathematics, numeracy or maths is, matters to me because it makes a difference to me, identifying as an adult numeracy teacher and teacher educator, to what and how I teach. However, this existential question is complex as teaching is bound up with social relations and political constructs. Therefore it is no use me knowing what I think mathematics, numeracy or maths are; it matters what the students, my colleagues, the teaching institution, the curriculum designers, the examinations boards and the governments education and skills departments think they are. As I have tried to show there are many positions to take and so we must be vigilant and ask questions about what others mean.

Why qualify mathematics?

I have focused on mathematics, numeracy and maths as the significant terms in this debate. However, many other terms have been used to qualify, describe, enhance or limit 'mathematics'. Usually these terms have been invented to focus on a particular use or assessment of mathematics, and the meaning is given by the context. Here are eight examples.

- Applied mathematics

- Basic mathematics
- Critical mathematics
- Ethno-mathematics
- Functional Mathematics
- Mathematical Literacy
- Quantitative Literacy
- School Mathematics

Perhaps one of these terms is more comfortable for you? Or is one you find most useful in the context in which you are working. When you use your chosen term is the meaning close to one or several of the descriptions of numeracy or mathematics given above? Why is it necessary for you to use this term? When is it necessary for you to use this term?

Questions need to be asked about what term is used to describe what we are teaching. They need to be asked about a new curriculum, about assessments, about international comparative tests, about education policies and about research questions. We need to ask:

- What term is being used?
- When is it used?
- Where is it used?
- About what is it being used?
- Who is using it?
- What is the purpose of using that term and not another?

Conclusion

This paper has continued the discussion I began in 2002 about the meaning of numeracy. In most situations, for most people, describing what is meant by numeracy is part of a process with a different goal. I have looked at what people have said about numeracy and compared them to each other and also compared them to what has been said about mathematics. The recent use of 'maths' in the UK has also been added to the discussion.

In the final part of the paper I have noted that many other terms have been, or are used when talking about mathematics or numeracy in different contexts. The discussion must continue with a series of questions that investigate the purpose of using a particular description in a particular context. Those answers will in turn need to be investigated.

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'Don't Ask Me about Maths – I Only Drive the Van'

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Abstract

The Mathematics experience in the classroom is disciplined by the curriculum and the expected learning outcomes, and taught with a terminal assessment in mind. Depending on the subject area and the course objectives, related topics are introduced in a sequence that suits the very practical considerations of the time available and the mixture of ability amongst the students and the teacher's commitment and capacity to engage with the subject. The level of sophistication of the mathematics increases incrementally from the fundamentals towards the most abstract concepts to meet the expectations associated with the calibre of the academic award sought. Such circumstances stand in stark contrast with the experience of mathematics outside the classroom, to the extent that many people perceive themselves as not having any use for, or facility with, mathematics of any kind. A recent National Survey in Ireland (Keogh, 2013) designed, to hear what people at work had to say about their use for mathematics, confirmed the use/denial paradox, i.e. that mathematics are generally important but personally irrelevant. Curiously, the same respondents self-reported high degrees of reliance on numerate behavior (Keogh, Maguire, & O'Donoghue, 2012).This paper recounts the numerate behaviour of a delivery van driver, and makes connections to the broad range of mathematics concepts upon which he relied and about which he was entirely oblivious. The key message in this paper is that work practice underpinned by mathematical thinking and behaving, however elaborate it might be, is dismissed routinely as something other than mathematics. Nevertheless, the opportunity to view his own set of mathematical knowledge and skills, however acquired, revised the van driver's opinion of himself, such that he invited the researcher to tell his wife that he was not stupid.

Keywords: adult numeracy, numerate behaviour, mathematics concepts

Introduction

In keeping with the theme of the conference, this paper draws a contrast between mathematics as it is taught and learned inside the classroom and its role in the outside world, where it may become enmeshed in the complex business of life, living, community and work. This is not to suggest that the classroom construct is any less 'real' than the so-called real world, but rather to underline that it is a different reality. A simple example may serve to illustrate. Tom's work requires him to visit a number

of customers that are scattered throughout Dublin's South Inner City. The quantity, variety and configuration of the customers change every day, as do their specific needs. The challenge is to do this as efficiently as possible to save time or money or to meet the conditions of some other operational metric. Inside the classroom, this is viewed as the classical Travelling Salesman Problem (TSP).

The 'Virtual' Travelling Salesman Problem

The 'virtual' TSP is the focus of the classroom. It is based on the conceit that a travelling salesman must visit a quantity of points, fixed in geography, by the most efficient tour; efficiency being a function of a common cost factor, e.g., fuel consumption, that is shared at least pairwise, Figure 1.
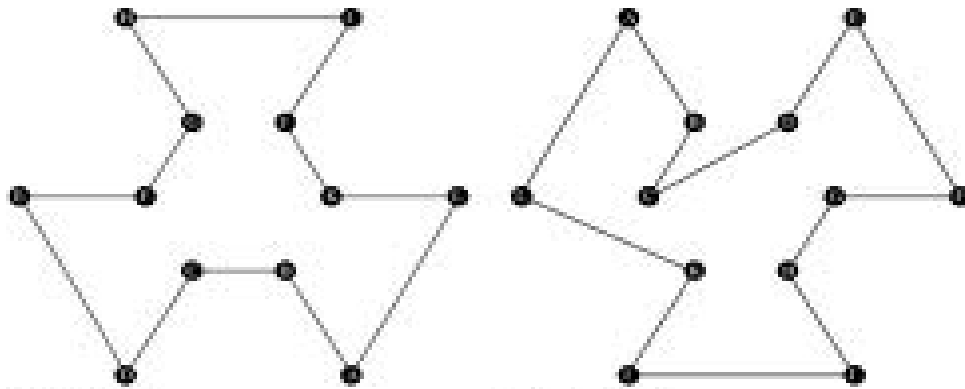
Figure 1. Alternative 'tours' visiting 12 points

Shown are two ways of touring 12 points, however, the number of possible solutions is given by the formula (n-1)! which is an almost inconceivable 40 million tours, or thereabouts. Typically, other methods are used to determine a set of better solutions, among which the easiest to understand is the 'greedy algorithm', guiding the salesman to the next nearest neighbour, although this approach is thought to be sub-optimal. An alternative is a 'brute force' attack, which is to try, and try again, avoiding sub-tours, or linking locations around which the points seem to cluster. That this type of problem may be more easily solved by computing, led to the introduction of Dynamic Programming, Figure 2.

$$
\begin{aligned}
\min \quad & \sum_{(i,j)\in E} c_{ij} x_{ij} \\
\text{s.t.} \quad & \sum_{j\in V} x_{ij} = 1 && \forall i \in V && (\text{assign\_i}) \\
& \sum_{i\in V} x_{ij} = 1 && \forall j \in V && (\text{assign\_j}) \\
& u_i - u_j + 1 \leq (|V|-1)(1-x_{ij}) && \forall (i,j) \in V, i \neq 1, j \neq 1 && (\text{mtz}) \\
& 2 \leq u_i \leq |V| && \forall i \in \{2,..,|V|\}, \\
& x_{ij} \in \{0,1\} && \forall (i,j) \in E
\end{aligned}
$$

Figure 2. An extract from the rationale that underpins Dynamic Programming

Dynamic programming builds on the Fibonacci Theory of sequences. It comprises an exhaustive cycle of calculations to determine the most efficient tour, depends on common 'cost' estimates and seeks to eliminate sub-tours. Whether this represents a useful support to the actual travelling salesman is not clear, for a variety of reasons.

The Actual Travelling Salesman Problem

Background

Tom works for a records management company, which stores up to 3 million items in a secure, fire-protected, purpose-built warehouse, with the promise to return them to the owner on demand. Every day, both in the morning and afternoon, Tom is provided with a bundle of up to twenty 'work orders' each detailing the workload for a specific customer.

Tom doesn't 'do' maths; he drives a van

Tom's first reaction is to assess the content of the work orders to determine whether he can pass on part of the work to a colleague on the basis of convenience. This is of limited benefit as it creates an informal obligation on him to reciprocate. Simply stated, Tom loads the van with items that have been placed in storage previously, and delivers them to order. Along the way, he must collect items from customers, return to base and unload the van, all the while adhering to the company's Standard Operating Procedures and local traffic regulations. Tom may not know the precise distance involved in a visit to each customer, because being confined to a relatively small part of the city, distance is not the primary concern. Instead there are multiple competing factors which interact in different ways, depending on the time of year, day of the week and time of day which add layer upon layer of complexity to his task. It is in Tom's interest to work efficiently, because he is not compensated for overtime. Superficially, there would not seem to be much evidence of numerate behaviour informed by mathematics knowledge, skills and competence, except for having to estimate the likely impact of a range of competing constraints.

Constraints

In deciding the optimal route, Tom takes account of traffic patterns that are affected by the school term, weather, temporary road closures, pedestrian ways, time-limited parking restrictions, road-works, accidents, demonstrations, parades, and garbage collections. Wet weather increases vehicle density, obscures visibility, and tends to clog the main arterial roads, causing Tom to choose alternative, minor roads, changing the delivery/collection sequence by bringing different customers into viable range. The side of the road on which a customer's premises is located, may influence the selection of the 'nearest neighbour' due to having to cross traffic or to navigate one-way traffic flow systems. The customer's readiness may cause him to adapt.

In addition, customers may not be ready to hand over their computer media backup at a time that suits Tom's optimum schedule. He is further constrained by having to observe the customer's standard operating procedures, their security arrangements, and parking facilities. Even the type of building can influence the time it takes to complete a delivery/collection, by whether it is equipped with an elevator. The loading sequence of the van has to be informed by Tom's solution to the TSP, such that the last items loaded will be the first to be unloaded, while allowing space for collections and load stability. Were the items in boxes to topple over, their contents could become intermingled creating a significant amount of otherwise unnecessary work for a colleague.

Tom's numerate behaviour informed by mathematics

On closer examination, Tom would seem to 'do' the following mathematics:

Space and Shape:
- Capacity, weight distribution, stability, identification, access to buildings

Chance:
- Traffic density, delay, time, opportunity, parking, late amendments, unexpected requests

Quantity and Number:
- Reconcile items listed on work orders to items collected or delivered, time calculations

Pattern and Relationships:
- Code recognition, sequence, security procedures
- Network mathematics

While it is relatively straightforward to identify the provenance of such workplace behaviour, a comparison of the contexts in which they are realised may be revealing, see Table 1.

Table 1.

Comparison of mathematics inside and outside the classroom

Comparison of Mathematics Inside and Outside the Classroom	
Inside the Mathematics Classroom	*Outside the Mathematics Classroom*
Complicated	Complex
Well defined content	Poorly defined content
Complete information	Incomplete information
Clear assessment criteria	Retrospective assessment
Consistent 'right' answer	Least 'worse' answer
Stable and predictable	Volatile
Low level risk	Potentially catastrophic risk
Personal responsibility	Material consequences for error

The term, 'complicated' is used here in the sense of being hard to understand, but completely replicable, whereas 'Complex' reflects the range of known and unknown ways in which components interact, with outcomes that are unpredictable and unlikely to be precisely repeatable.

Classroom content is defined by curriculum, complete within its own parameters, stable and consistent, with associated assessment mechanism, whereas the World outside is seldom so. Participating in a mathematics class carries low levels of risk, with most of the responsibility for the expected outcome being borne by the student and the extent of his/her engagement, and persistence. This stands in stark contrast with the world of work especially, where risk and consequences can have a profound effect on livelihood and live itself.

Implications for Mathematics Teaching and Learning for Adults

The normal use of the term 'transfer' connotes a transplanting, or the act of moving someone or something to another place. There is no embedded implication for the need for change, only relocation. In this sense, the expectation of successful transferability of knowledge skills and competence from its source to destination may be problematic (Evans, 2000). The context of the mathematics classroom comprises a unique set of individuals, each unique in their own right, occupying different roles across a landscape of personalities, not simply learner or teacher, surrounded by a single discipline in which people are motivated to participate for multiple outcomes and regulated by formal and informal rules. Similarly, any context outside the classroom may be characterised in multiple dimensions, and possibly the intersection of multiple contexts. In this light, the idea of a simple transfer of knowledge, skills and competence seems very unlikely. A more realistic expectation may be the possible integration of mathematics knowledge skills and competence re-contextualised taking account of the non-classroom paradigm.

To this end, the classroom pedagogy may need to recognise that practical constraints may apply outside the classroom that may be inconvenient when exploring a mathematical principle.

Furthermore, and especially for adult learners, it should not be beyond the competence of the teacher to explain the purpose of mathematical thinking and algorithms, other than to pass an examination. Failing that, the rules and symbols of mathematics may amount to little more than a series of 'tricks' that amuse only the initiated, and largely irrelevant otherwise.

It may be of longer term benefit to equip students with the tools and techniques to analyse the context into which they may be expected to realise their mathematics knowledge and skills.

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Adult Students' Experiences of a Flipped Mathematics Classroom *

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Abstract

The flipped classroom is a flexible blended learning model that is growing in popularity due to the emergent accessibility to online content delivery technology. By delivering content outside of class time asynchronously, teachers are able to dedicate their face to face class time for student-centred teaching approaches. The flexibility in implementation of a flipped classroom allows for a diversity in student experiences. The study presented in this paper uses qualitative methods of analytic induction to conduct a case analysis on survey and interview data collected from students participating in a flipped adult mathematics upgrading course at an urban Canadian university near Vancouver, BC. The key phenomenon of interest in the study is how adult students experience a flipped mathematics classroom. Of secondary interest is how factors such as autonomy and goals interrelate with these experiences. It is found that flipped classrooms can bifurcate into two types of student interaction: completely engaged and self-paced. Key interrelated factors in this bifurcation include adoption of cognitive autonomy support, goal orientation, and attendance.

Keywords: flipped classroom, autonomy, goals, classroom experience

Introduction

Empowering adults to learn mathematics, especially when they have encountered low mathematical performance in their past and have returned to the subject for the key purpose of obtaining high school prerequisites required towards a new career path, can be very challenging. The underlying goal of this study is motivated by the desire to enrich the experiences of this population of adult learners by providing them with a student-centred learning environment, which differs from the dominant teacher-centred learning environments they were most likely exposed to in their public school experiences.

In a teacher-centred learning environment, the focus is on pursuing the teacher's agenda, which is not directly related to emergent student learning needs. In contrast, student-centred learning approaches focus on the student and their learning journey. The notion of a student-centred learning environment is rooted in constructivism and embraces student agency. Knowledge is actively constructed by the learner rather than imparted by the teacher, and "goals are negotiated and selected by the learners" (Elen, Clarebout, Léonard, & Lowyck, 2007, p. 107). In this research, Elen et al.'s (2007) transactional view of student-centred learning is adopted, where there is a "continuous interchange between students' and teachers' responsibilities and tasks" (p. 108). The key premise is that the teacher observes student interactions and adapts teaching interventions accordingly to student needs.

Overall, student-centred approaches have been found more effective than teacher-centred ones (Åkerlind, 2003; Barr & Tagg, 1995; Grubb & Associates, 1999; Grubb & Cox, 2003; Kember & Gow, 1994; Prosser & Trigwell, 1999). However, creating student-centred learning environments can be challenging for teachers, especially in mathematics, where curriculum constraints are demanding. Wang (2011) notes that "student-centred teaching tends to be more time-consuming and unpredictable than whole-class lecturing" and that "teachers working under a fixed curriculum and schedule are inclined to organize the class in a more teacher-centred manner to secure the completion of required

tasks" (p. 157). In an effort to relieve these tensions between allowing for student-centred learning practices and maintaining adherence to the curriculum, educators have become drawn to the affordances provided by increasingly accessible technologies to deliver content asynchronously out of class time while dedicating class time for student learning. Bergmann and Sams (2012) have coined the phrase 'flipped classroom' in reference to this teaching approach.

The concept of reversing content delivery and practice time is not a new phenomenon in education, but the increasing accessibility to technology that allows teachers to create their own content videos and the improved ability available for teachers to share their teaching practices to a wider audience online have contributed to the increasing popularity of the flipped classroom model. Media outlets such as USA Today (Toppo, 2011), Washington Post (Strauss, 2012), and CNN (Green, 2012) have covered experiences and opinions regarding the flipped classroom. However, research based literature pertaining to flipped classrooms is still limited. Several studies report increased student achievement in flipped classrooms (Day & Foley, 2006; Green, 2011; Johnson, 2013; Kirch, 2012; Mussallam, 2010), but few of them relate directly to a mathematics context, let alone the adult population.

The most notable studies within a mathematics context focus on student perceptions. One of these studies looks at an undergraduate level statistics course (Strayer, 2008) and the other looks at a set of high school level mathematics classes (Johnson, 2013). Strayer (2008) compares student responses from a flipped classroom version of an undergraduate statistics course with that of a traditional classroom version of the same course. He finds that students in a flipped classroom can experience higher levels of innovation and cooperation than those in a traditional classroom but that they can also experience feelings of unsettledness due to the unpredictability of class time. Students in the flipped classroom can also find the learning model difficult to accustom to if they are used to a traditional approach. In contrast, Johnson (2013) finds that high school mathematics students experience the flipped classroom approach more positively. His students evidence enjoyment from classroom learning activities, frequent interaction with teacher and peers, and a reduction in time necessary to spend on homework outside of class time. Johnson (2013) also finds evidence of improvement in students' perceptions of engagement, communication, and understanding. The varying and almost contradictory results in these studies may in part be attributed to various methods of implementation and a difference in student population. Based on these two small-scale studies, one could conclude that adult learners may have a more difficult time adjusting to the teaching approach than high school students. However, the evidence for such an argument is not substantive enough and needs further exploration.

More importantly, there is no single method of implementation of a flipped classroom, and just like with any student-centred teaching approach, its success rests on a teacher's pedagogical sensitivity and ability to adapt to student needs. Although student-centred approaches are desirable, they are not always easy to carry out. The flipped classroom approach provides teachers who want to evolve their classes into student-centred learning environments with the option to deliver direct instruction outside of class time, leaving time during class for student-centred tactics. Flipped Learning Network (2014) has defined flipped learning as "a pedagogical approach in which direct instruction moves from the group learning space to the individual learning space, and the resulting group space is transformed into a dynamic, interactive learning environment where the educator guides students as they apply concepts and engage creatively in the subject matter" (para. 2). They claim that flipped classrooms can lead to flipped learning through a flexible environment, a rich learning culture, intentional content, and a professional educator, but that a flipped classroom in itself does not promise flipped learning. Rather, a flipped classroom offers teachers a means with which to employ student-centred approaches. Kachka (2012) notes that "the increase of teacher-student interaction during class time is what characterizes [the flipped classroom model's] success" (para. 6). Student-centredness within a flipped classroom, by its nature, affords student autonomy over learning, and is closely tied with factors such as goals, self-efficacy, and anxiety.

The research presented in this paper is therefore motivated by the question of how adult mathematics students experience a student-centred flipped classroom environment that offers opportunities for student autonomy over learning in the context of an adult mathematics high school level upgrading course at the University of the Fraser Valley in British Columbia, Canada. A secondary question that

guides the study pertains to how factors such as goals, self-efficacy, and anxiety are interrelated with adult student experiences in this classroom. For purposes of brevity, only factors of autonomy and goals are detailed in this paper. In what follows, key literature, background, results, and conclusions from the study are overviewed as pertaining to autonomy and goals. Possible implications for the field of mathematics education are also discussed.

Key literature

Analysis of the context of this study is informed by literature on the provision of autonomy support. Student experiences in the context are examined in relation to literature on goal orientation. These theories are briefly overviewed in order to ground the results.

Autonomy

Student-centred learning environments by nature allow for student autonomy. In general, autonomy is viewed as the availability of choice, which is evident in Black and Deci's (2000) definition: autonomy is supported by providing students with "pertinent information and opportunities for choice, while minimizing the use of pressures and demands" (p. 742). Studies have shown that students of autonomy supportive teachers experience more classroom engagement, positive emotion, self-esteem, creativity, intrinsic motivation, psychological well-being, persistence in school, academic achievement, and conceptual understanding (Assor, Kaplan, & Roth, 2002; Benware & Deci, 1984; Deci & Ryan, 1985, 1987; Deci, Nezlek, & Sheinman, 1981; Hardre & Reeve, 2003; Koestner & Ryan, 1984; Reeve & Jang, 2006; Reeve, 2009; Ryan & Grolnick, 1986; Vallerand, Fortier, & Guay, 1997). Therefore, it is important to consider the role of autonomy and its implications for mathematics classrooms.

Although positive effects are associated with autonomy in classrooms, it is important to emphasize that autonomy cannot simply be provided, it needs to be supported. Autonomy supportive teaching should "adopt the students' perspective, welcome students' thoughts, feelings, and behaviours, and support students' motivational development and capacity for autonomous self-regulation" (Reeve, 2009, p. 162). Stefanou, Perencevich, DiCinto and Turner (2004) classify autonomy support into three dimensions: *organizational* autonomy support, *procedural* autonomy support, and *cognitive* autonomy support.

Organizational autonomy support allows students to control their environment by directing them to choose classroom rules, the pace at which they learn, due dates which they set, students with whom they work, and ways in which they are evaluated. Meanwhile, *procedural* autonomy support allows students to control the form in which they present their work by inciting them to choose materials they use for a project, the ways in which they display work, and the ways in which their materials are handled. Finally, *cognitive* autonomy support allows for students to control their learning by encouraging them to generate their own distinct solutions, justify their solutions according to mathematical principles, evaluate their own work, evaluate work of their peers, discuss multiple approaches, debate ideas freely, ask questions, and formulate personal goals.

Stefanou et al. (2004) argue that although organizational or procedural autonomy support may be necessary, it may be insufficient in maximizing motivation and engagement. They claim that cognitive autonomy support is the most essential type of autonomy support in order for positive educational benefits such as motivation and engagement to occur. Although Stefanou et al. (2004) do not clearly indicate whether organizational and procedural dimensions are best structured or left autonomous, a study conducted by Jang et al. (2010) suggests that student engagement can be more prominently observed when a learning environment has higher levels of structure (i.e. structured organizational and procedural dimensions) as long as students are provided with high levels of cognitive autonomy support. They note that structure should not be confused with control. Even when a dimension is more structured than autonomous, the teacher should maintain respect for student thoughts, feelings and actions within the structure. Although the necessity of organizational and procedural autonomy support is not clearly defined in the literature, there is consensus with regard to the importance of cognitive autonomy support in relation to heightened student engagement and motivation.

Goals

Additionally, goal orientation can have a positive influence on performance and motivation in the face of a challenging task, such as that of learning mathematics (Grant & Dweck, 2003). The predominant view of goals that informs analysis in this study is that of Achievement Goal Theory. This theory is rooted in the belief of intelligence as being either fixed or malleable giving rise to either *performance* (self-enhancing) or *learning* (mastery) goal orientations, leading to various motivation driven behaviour patterns that depend on self-efficacy beliefs (Dweck, 1986; Pintrich, 2000).

In a *learning* goal orientation, "individuals seek to increase their competence, to understand or master something new" whereas in a performance goal orientation, "individuals seek to gain favorable judgements of their competence or avoid negative judgments of their competence" (Dweck, 1986, p. 1040). Grant and Dweck (2003) provide evidence that a learning goal orientation positively affects performance and motivation in the face of challenge while the performance goal orientation only positively affects performance and motivation if no challenge is present. In extension of Dweck's (1986) theory, Dupeyrat and Mariné (2005) discover that for adults returning to school, "mastery [or learning] goals have a positive influence on academic achievement through the mediation of effort expenditure" (p. 43).

Further, Hannula (2006) shows evidence that "students may have multiple simultaneous goals and [that] choices between them are made" (p. 175). He claims that motivation is structured through the mediation of needs and goals with emotions and that a desired balance of goals can be promoted by offering students a safe learning environment that focuses "on mathematical processes rather than products" (Hannula, 2006, p. 176). Such an environment can be created through the provision of cognitive autonomy support and is possible within a flipped classroom context.

Background

In what follows, the context of the study and the methods used to collect and analyse data are overviewed.

Context

The context of this particular study is a full-term 60 hour adult mathematics upgrading course referred to as Math 084 offered through the Upgrading and University Preparation Department (UUP) at the University of the Fraser Valley (UFV). UFV is a fully accredited public multi-campus university primarily located in the Fraser Valley just east of Vancouver, British Columbia, Canada. The UUP department at UFV offers programmes in Adult Basic Education (ABE) for adults of all backgrounds and ages who want to meet their educational goals such as completing prerequisites for post-secondary programmes, earning the BC adult graduation diploma, or improving skills for personal benefit.

Math 084 serves as a requirement for the Dogwood Diploma (graduation diploma in British Colombia) and is the first out of two courses that together serve as a prerequisite for most undergraduate programmes that lead to career paths such as teaching, nursing, business diplomas, etc. The course covers a variety of topics including linear equations, linear inequalities, quadratic equations, radical equations, operations with polynomial, rational, and radical expressions, and function graphing. It is traditionally taught with 60 lecture hours and 30 individual or group work hours, which makes it a primarily teacher-centred learning atmosphere.

In contrast, the flipped classroom implementation of Math 084 fostered a student-centred learning atmosphere. Video lecture lessons[8], online quizzes, announcements, and practice problems were posted in an online learning management system (i.e., Blackboard Learn), and students were asked to preview this content prior to class as homework. More importantly, having the content available online afforded time during class for student-centred content-related discussions, group learning activities, practice time, and assessments. This means that the class design was aligned with the tenets of the Flipped Learning Network's (2014) description of flipped learning, which may emerge within a flipped classroom. Classes typically consisted of approximately 80 minutes of teacher facilitated

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[8] Videos can be viewed by visiting Judy Larsen's YouTube Channel.

discussions and/or group learning activities and 80 minutes of time for completing assignments. This means that classes were facilitated by the teacher, who decided on which activities to initiate based on their interpretation of student needs.

An example of an open ended group learning activity problem facilitated by the teacher during the course is the National Council of Teachers of Mathematics (2008) Barbie Bungee Activity. During this activity, students were asked to find the maximum number of rubber bands required to allow a Barbie doll to 'bungee jump' from a certain height without hitting her head. Students, in random groups, were given rubber bands and a doll and were asked to make the prediction for the number of rubber bands required. Eventually, through discussion, students noted the linear relationship between the number of rubber bands and the measure of the doll's descent. This led to further discussion on linear equations and slopes.

Another instance of an activity facilitated during the course is that of student-generated examples[9]. This is not referring to Watson and Mason's (2005, 2002) concept development approach to learner-generated examples, but rather the opportunity for students to generate examples for purposes of involvement in the learning process. One instance of a student-generated example activity is when students were provided with a collection of 3-dimensional geometric objects and were asked to build a new object composed of two or more smaller objects. They were then asked to give their new composite object to another group to find the surface area and the volume of the given composite structure. This activity led to some interesting discussion and even a Google search regarding the surface area of a cone because it was not provided in the course textbook. Yet another case of a student-generated example activity is when students were asked to use whiteboards to develop exponential expressions that needed simplification. They were then asked to pass their problems to another group for simplification. Interesting examples arose from such activities. One example in particular was that of a student who created a complicated exponential expression, but created it so that the entire expression was taken to the power of zero indicating that the student understood the implication of a power of zero (See Figure 1 below).

Figure 1. Student generated example 1.

Other group learning activities consisted of group concept review sessions. For example, students used whiteboards to develop reasoning for why certain properties exist, such as the rules for simplifying exponential expressions. Products from review sessions were often documented with a camera and posted on the course website to help provide study materials for students in preparing for tests.

Equally important to the choice of activities in the promotion of engagement and understanding was the method of grouping students so that they would productively collaborate. Liljedahl (2014) asserts that visibly random groups lead to positive observable changes such as "an elimination of social barriers, [an increase in] mobility of knowledge between students, [a decrease in] reliance on the teacher for answers, [and an increase in] engagement" (p. 130). During the first half of the term,

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[9] Student-generated examples are used colloquially here in the sense that students were asked to generate examples for the purposes of assessment or engagement and not in the more defined sense that Watson & Mason (2005, 2002) indicate in respect to constructive concept development.

students were always grouped together randomly to increase the likelihood of students working with as many other students as possible in alignment with Liljedahl's (2014) suggestions for student grouping. Eventually, students found their favourite peers to work with and they settled into preferred groups.

Anything that contributed to a student's final grade (assignments and tests), with a few exceptions, was completed and submitted during class time. In essence, the in-class workload and the out-of-class workloads were swapped or flipped as compared to a traditional class. Most importantly, class time provided students with opportunities to engage with colla-borative problem-based learning tasks, a facilitative teacher, and a variety of learning tools.

Method

The Math 084 flipped classroom outlined above was implemented during the Winter 2013 term. The course started with 25 total students enrolled, 18 of whom completed the course. It should be noted that low completion rates are very common in these courses and many students often stop showing up due to life circumstances. Out of the 18 students who completed the course, two were registered, but were completing the course at a distance, and therefore were not part of the flipped classroom experience. This leaves 16 students who experienced the flipped classroom throughout the entire term, 14 of whom gave consent to participate in the research study. All 14 of these students appeared to be in their twenties and were completing the course either to satisfy prerequisites towards career-driven programmes or to complete their high school diploma.

Data collected consisted of observational data, interviews, and surveys (including in-class surveys and follow-up email surveys). As researcher and instructor of the course, I collected observational data throughout the term in relation to classroom interaction, goal statements, self-efficacy, anxiety, etc. and tabulated each observation into an Excel spreadsheet document for analysis. Interview and survey data was collected by an external co-investigator during the term while I was away from the room in compliance with local research ethics requirements. After final grades were posted, I was given access to all data collected by the external co-investigator.

Analysis of data was performed according to the tenets of analytic induction, a qualitative method of analysis rooted in grounded theory. Much like in grounded theory, the inductive analyst recursively codes the data looking for themes to emerge; however, analytic induction allows for an *a priori* proposition or theory driven hypothesis to be used as a lens to deductively analyse the data in contrast to grounded theory, which begins inductively through open coding (Glaser & Strauss, 1967, cited in Patton, 2002). In this research study, the key *a priori* theory used in the deductive phase of the analysis was that of Stefanou et al.'s (2004) distinction between types of autonomy support. Other theories used in the analysis pertained to goals, self-efficacy, and anxiety in the context of mathematics education (Ashcraft, 2002; Bandura, 1997; Biggs, 1985; Dweck, 1986; Hannula, 2006; Jang et al., 2010; McLeod, 1992; Zimmerman, 2000).

Prior to the theory driven deductive phase of analysis, a preliminary analysis of data was performed to draw out data relevant to the goal of this research, which is to characterize student experiences in a flipped classroom. During this preliminary investigation, it quickly became evident that there were three levels of student interaction in the class. The class design provided students with a diversity of learning tools, and although most students utilized all learning tools during the first part of the term, they eventually gravitated towards certain learning tools as they pursued completion of the course. In particular, some students chose to utilize class time completely in order to gain better understanding of topics. These students willingly participated in all classroom activities. Others chose to focus more on out-of-class learning materials such as the online videos and the textbook. These students tended to attend less regularly or chose to opt out of activities offered during class time. There were also those who shifted between these types of interaction throughout the term.

For each of these three types of interaction, two participants whose actions were reflective of each of these types of interaction were carefully selected. This means that the six participants selected as cases consisted of two students who participated completely in both in-class and out of class components of the flipped classroom (Group 1), two students who at first participated completely with the flipped

classroom model but later fell behind and chose only to participate in out of class components (Group 2), and two students who tried participating in the flipped classroom model completely, but quickly participated only in what was absolutely required in the course (Group 3). These cases are summarized in Table 1 below.

Table 1.
Grouping of Cases

Group 1	Students who completely engaged in both in-class and out-of-class components.	Alexa (A)
		Kristy (A+)
Group 2	Students who at first engaged in both in-class and out-of-class components, but chose to opt out of class time activities near the end of the term.	Mark (A+)
		Ryan (A-)
Group 3	Students who tried engaging in both in-class and out-of-class components, but as soon as they could opt out of the activities, they did.	Lindsay (B+)
		Vanessa (A-)

Note.   All names are pseudonyms to maintain anonymity.

These cases were reflective of the three types of interaction in the course because out of the 14 participants, five were categorized as Group 1, five were categorized as Group 2, and four were categorized as Group 3. Further, grades obtained by these cases were within the grade range obtained by the majority of the students in the class (11 out of 14 students attained a B+ or higher and no students completed the course with a grade lower than a B-).

The data related to these six participants was aggregated to form cases that reflected various student experiences in the course. Each case was then analysed and coded according to the key a priori theories of autonomy, goals, self-efficacy, and anxiety in the context of mathematics education. This case analysis was followed by a cross-case analysis that inductively derived common themes across the data. As previously noted, the scope of this paper has been limited to only factors of autonomy and goals in order to provide adequate depth and detail.

Results

Results are presented by providing sample case data from the study as well as a cross-case analysis of key factors that are of focus in this paper: autonomy and goals.

Cases

In what follows, three cases are briefly outlined to provide samples[10] from each of the three groupings described in the previous section: Kristy (Group 1), Mark (Group 2), and Lindsay (Group 3). These are chosen for their strength in presenting key issues resulting from the study pertaining to autonomy and goals in a flipped classroom environment.

Kristy

Kristy was selected as a case in Group 1 because of her complete engagement in both in-class and out-of-class activities. She attained an A+ in the course and serves as an example of someone who experienced the flipped classroom to the fullest extent. Kristy attributed her success in the course to several factors including the ability to progress through lectures at her own pace, the time available to discuss concepts that were troubling during class, and the opportunity to teach others in the class. Although she was initially shy and nervous about being in the course due to her past negative experiences with mathematics, she soon found the learning environment comfortable and conducive to learning. She claimed in an initial survey that "up until this term, [she had] never liked mathematics and never grasped the concept." She noted that in high school, she kept falling behind with notes, didn't receive enough individual attention and was not shown things in a kinaesthetic manner, which resulted in poor achievement. Although she initially expressed concern about doing things the "right" way during classroom activities, she soon discovered that seeing multiple approaches is beneficial to

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[10] For complete case data, see Larsen (2013).

understanding the concept. She summarized her engagement in classroom activities on the follow-up survey:

> Although I want to say that the at home lectures were the most valuable part of the class, the group activities played an equal role in how well I learned the mathematical concepts. Being forced (I use the term lightly) into group activities during class allowed me to get to know my classmates, which made me feel a lot more comfortable asking any questions I had. Secondly, the other ideas and approaches that students had towards problems allowed me to see different ways of understanding the questions and different techniques to use when finding an answer … The way I see it is, solving a problem is great, but being able to explain how the problem works means you truly understand it. I was almost testing myself by teaching others. It became another way of studying for me … I was beginning to "know" math. I was starting to truly understand the concepts because I was able to study as much as I needed to since the lectures were always available to me.

(Follow-up survey)

In addition to her in-class engagement, her out-of-class engagement was also notable. On a survey taken at the middle of the term, she responded that outside of class time, she watched all of the videos (sometimes more than once) and took detailed notes from them. She also re-watched the videos if she was stuck on a concept, and if she couldn't figure something out, she made note of it and moved on knowing that she could ask about it during the next class. She made decisions about how much practice she needed to complete on each section claiming on a survey during the middle of the course, "If I feel strong on a concept I don't do all the examples and if I feel weak I do more than the given" (Week 8 survey). She showed appreciation for the videos on the follow-up survey:

> Having the lectures in video form allowed me to study them at my own pace and take notes a lot more accurately. The option of being able to pause or rewind the video instead of asking the instructor to stop or repeat was great as well since it does not stop anyone else's learning process.

(Follow-up survey)

During an interview, Kristy noted, "I feel like I'm walking out of the classroom knowing something, I'm not just wasting my time trying to get a letter grade, I'm actually taking something away from the class too" (Week 3 interview). By attending almost every class, participating completely in all in-class tasks, leading group discussions, and working on material at home frequently, she was able to develop a new interest in mathematics.

In summary, Kristy participated completely in all components of the flipped classroom throughout the entire term. However, she may not have been as successful in the flipped classroom environment had there not been initial organizational structure and continued cognitive autonomy support provided. She noted that she greatly appreciated being placed in random groups at the beginning of the course (an organizational structure) because otherwise she would have been too shy to communicate with others. At first, she was also uncomfortable with not knowing how her assignment was supposed to look, indicating a resistance towards procedural autonomy support. She also reacted negatively when I probed her to think on her own, indicating an aversion towards cognitive autonomy support. However, this quickly diffused as she participated in the course consistently and completely. Eventually, she came up with her own ways of solving problems and taught others comfortably, indicating that she found use in the cognitive autonomy support that was being provided as part of the classroom culture. Even though she entered the class wanting to satisfy prerequisites for a programme path, indicating a performance goal orientation, she managed to cultivate a learning goal orientation within the flipped classroom environment. Given that Kristy decided to continue in the course after finding out that she no longer needed the course as a prerequisite, the flipped classroom was a truly empowering experience for her.

Mark

Mark was selected as a case in Group 2 because he first engaged in both in-class and out-of-class components, but chose to opt out of class time activities near the end of the term when he wanted to get farther ahead with the material more efficiently. Mark attained an A+ in the course and showed complete interaction with the flipped classroom during the beginning of the term, but became more motivated to work individually after missing a few classes in the second part of the term due to a bad

case of the flu. His favourite part about the flipped classroom model as stated on the follow-up survey was that he could "come to class with questions and actually get the questions answered instead of being stuck out of class time." He also noted on this survey that he appreciated the freedom he had to learn content at his own pace and out of class time. From the beginning of the term, Mark showed inquisitiveness and engagement. He noted on his initial survey that he chose to take Math 084 "to get a better understanding of Math" because he is "just fascinated by how it works." Even though he completed Math 11 and 12 in high school eight years ago, he noted that he did not find it enjoyable at the time and he found that he had forgotten too much of it when he recently attempted to complete a first year calculus course. This informed his choice to take Math 084.

As mentioned, after a series of absences due to being sick in the latter part of the term, it was observed that Mark began to opt out of activities and worked on his own in the back of the classroom. During these times, he took the liberty to choose when to engage in the entire class and when to engage in his own work. He did this by looking up when something interesting was happening and looking down at his work when he felt he didn't need to be paying attention. In a survey completed near the end of the term, he noted, "I used class time more for doing homework [as the term progressed] so that I could ask questions." He clarified this later claiming, "Near the end of the term, the topics we were doing I was very familiar with and I wanted to get ahead on my homework so that I could go back and check and think of any questions I could ask before the final" (Follow-up survey). He was actively engaged in course content out of class time throughout the term and was able to develop his own method of studying for a test by taking questions from each section and making mock tests for himself.

To showcase Mark's search for understanding, it is worthy to mention his classroom interactions during the first half of the term. First of all, Mark consistently asked questions that demonstrated his desire to test his own conjectures and search for generalisations. One example of such a question, noted in the observational data, was when he inquired about whether there existed a general method for finding the domain and range of any function after he determined the domain and range for a few rudimentary functions. Secondly, Mark was often observed attempting to complete activity problems in several different ways and working collaboratively with others, encouraging them to think in various ways. In his follow-up survey, he noted that his favourite type of activity was "one that allows you to come to the same solution but with multiple paths." Based on my observations, he thrived within activity problems that were open-ended because he worked towards creating difficult scenarios in order to challenge himself. One example of this was when he created a very complicated three dimensional shape consisting of a cone nested within a cylinder with a half-sphere on top (See Figure 2 below). He then encouraged his group to figure out the volume and surface area of the shape. We hadn't learned how to find the surface area of a cone, so it led the class to learn more than was expected. Combining several shapes also gave students the opportunity to learn how to alter formulas they had learned.

Figure 2. Student generated example 2.

Mark was also interested in developing reasoning. In a survey during the early part of the term, Mark reflected on an activity that asked students to justify reasoning for various exponent rules on the

whiteboards in groups. He claimed that the activity was "very helpful in understanding the way rules for exponents work instead of just memorizing them" and that that is his "favourite way to learn things" (Week 3 survey).

In summary, Mark participated in all components of the flipped classroom until about two thirds of the way through the course, when he began to opt out of class time activities. Interestingly, Mark exhibited a learning goal orientation right from the beginning with his original intent for taking the course being to get a better understanding of mathematics. During the beginning of the course, he readily communicated with others and was intrigued by the activities, using all tools that were available to him. He participated in the classroom culture by proposing interesting ideas to others and helping them with their work. Based on the examples provided of his interactions in the problem solving activities, it is evident that Mark embraced cognitive autonomy support during the first part of the term. Additionally, his increasingly independent thinking and learning throughout the term contributed to his ability to make good use of the organizational and procedural autonomy support that became increasingly available. After being sick for a while and being away from class, he began to come to class without engaging in classroom activities. Due to his absences and low classroom involvement, I perceived his actions as that of someone who had fallen behind in his work and needed to catch up. However, Mark was actually moving ahead. He wanted to learn further material, engaging autonomously with it, so that he would know what to ask questions about. However, as the term neared completion, time constraints seemed to alter his goals. He began to participate less and less in the classroom activities, and although his goals were still predominantly of learning, he showed goals of performance in his expressions of concern around completing course requirements. Overall, Mark's experiences with the course were very positive because even when he was sick and had to miss class, he was not greatly inconvenienced by it because of the accessibility of learning materials.

Lindsay

Lindsay was selected as a case in Group 3 because although she initially tried engaging in both in-class and out-of-class components, she soon opted out of class activities after falling behind with the material and realizing that the activities were not required towards course completion. Remarkably, even though it was noted that she was absent a lot during the last third of the term, she was able to complete the course with a B+ by watching the videos, completing examples from the videos, and completing assigned graded textbook problems. Although Lindsay engaged in the course in an individual manner, it proved to be more beneficial for her than another completely individually paced course she had previously taken because she had a greater variety of resources available. Lindsay also noted on the follow-up survey that although her primary goal with Math 084 was to get a good grade and complete her prerequisite requirements towards an animal health technician program, the flipped classroom environment was beneficial for her because it helped her learn how to ask questions and provided her with enough material out of class time to work through and catch up with when she fell behind.

Lindsay particularly enjoyed learning from the videos out of class time because she was able to "go through [each video] slowly and do the example questions one step at a time" (Week 8 survey). She also noted that she really appreciated the opportunity to "pause and rewind the video whenever" she needed to (Week 3 survey). At the end of the term, Lindsay wrote, "The ability to watch lessons at home and at [my] own pace was probably the thing I liked the most about the class" (Week 14 survey). On the Week 8 survey, she noted that she watched every video in great detail, took notes from the videos, and paused the videos in order to try the example questions on her own before proceeding with the video. She also claimed on this survey that she referred to textbook examples often and tested her understanding by completing the online quizzes. It was observed that when she didn't understand a concept well, she gravitated towards re-watching the videos before asking any questions.

Lindsay tended to work individually and as a quiet observer during class time. She tried engaging in the group activities during the first third of the course, but always seemed overwhelmed in the group setting. When in her proximity, she would often ask me probing questions seeking confirmation of the work her group was doing. During the latter part of the course, it was observed that Lindsay began to use class time even more individually. As the material became more difficult, Lindsay began to be absent more often. She soon fell behind with the material and began to treat the flipped classroom as a

place to learn individually. During one set of consecutive absences, she emailed me explaining that she needed to stay home because she wanted more time to go over the videos and complete missing graded problems. It was evident that she was avoiding group work and desired to complete course requirements as efficiently as possible. This is evidenced in the following survey response:

> One thing I didn't really like was the amount of group work we had to do. Sometimes it was helpful but sometimes it seemed to complicate things … [As the term progressed], I used class time to hand in work, work on graded problems and do tests. I [made] sure when I [got] stuck on something to ask for help.
>
> (Week 14 survey)

However, on a survey taken during the middle of the term she wrote, "This class has helped me realize that asking for help more when I need it is OK" (Week 8 survey). During the latter part of the term, she watched the videos in great detail and then came to class to clarify concepts that she struggled with. I observed that most of her clarifications pertained to implementation strategies of the various procedures outlined in the videos and used in the textbook. These clarifications were very important for her.

In summary, although Lindsay tried to engage in all components of the flipped classroom, she quickly began to avoid components that expected her to adopt cognitive autonomy support. Upon entering the class, Lindsay held a strong performance goal orientation with her main reason for engaging in the class being to satisfy a career prerequisite. The organizational structure of requiring students to work in random groups at the beginning of the term allowed her to experience a classroom culture of learning. However, during the times when she was asked to work with others, she tended to observe the others in the group rather than initiate discussion. She seemed to resist cognitive autonomy support within problem solving opportunities and often became confused by other students' approaches to solving problems. This was at times frustrating for her and it may have interfered with her performance goal orientation because it compromised the efficiency of learning the material. As soon as more organizational and procedural autonomy support was available, she chose to focus on the videos as her main learning tool and was grateful for their accessibility. When she was behind with the material, she did not feel adequately prepared to participate in group activities, causing her to avoid class time. Although she missed a lot of class time in the second half of the course, she was able to complete the course successfully due to the availability of the online videos. The flipped classroom seemed to be beneficial for her because as she noted, it helped her learn how to ask questions.

Cross-case analysis

The aggregated case data and case by case analyses of all six cases in the study evidenced a bifurcation in how participants experienced the flipped classroom during the second part of the term once students became accustomed to the class structure. A cross-case analysis clarifies that the bifurcation was made possible, in part, by the autonomy support provided in the structure of the Math 084 flipped classroom's learning environment. Student goals were interrelated with the bifurcation, and attendance surfaced as an emergent theme. These results are overviewed in the subsequent paragraphs.

Autonomy

The flipped classroom, as implemented in this study, offered students an opportunity for autonomy by allowing them to engage in a variety of components: learning activities, classroom community, and accessible learning materials. Most importantly, cognitive autonomy support was provided during class. This can be seen in Alexa's survey response:

> If I was confused about anything, we would explain everything in great detail and have debates about it … I learned different ways to solve problems during the activities and others learned from me. This was fantastic.
>
> (Follow-up survey)

At first, all students participated in all components of the class when procedural and organizational structure was provided in an autonomously supportive way, through the use of random groups, due dates, specified assignment submission procedures, etc. As the term progressed, more autonomy

support was provided over procedural and organizational dimensions in the class. Simultaneously, a bifurcation of student experiences occurred. In particular, election of cognitive autonomy support began to change. Figure 3 below serves as a subjective visual representation of students' expressed desires for either high cognitive autonomy, occasional cognitive autonomy, or no cognitive autonomy as coded from the case data in relation to the time in the term.
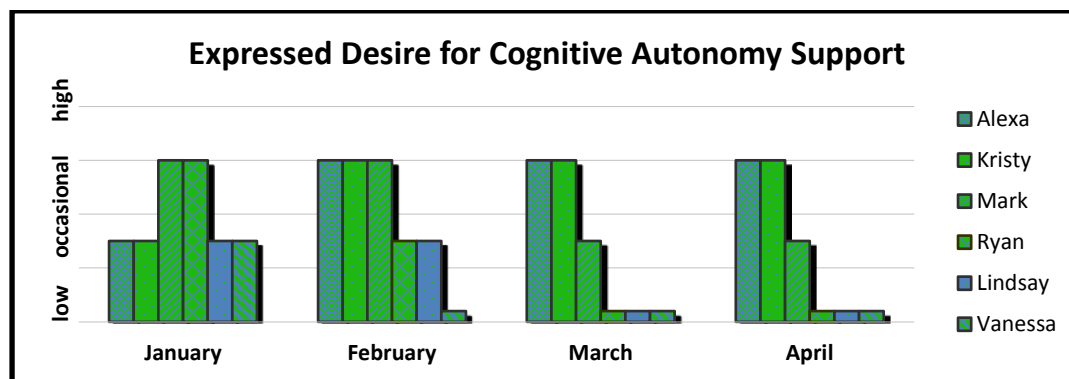
Figure 3. Cognitive autonomy over term.

When procedural and organizational autonomy support was provided during the latter half of the term, students split into those engaging in the flipped classroom completely and those interacting with it in a more or less self-paced manner by either opting out of classroom activities or choosing to not attend class.

Falling behind seemed highly associated with absence. Some students fell behind because of external factors which influenced absences (Mark and Ryan), while others chose to be absent because of internal factors such as falling behind (Lindsay). Falling behind can be extremely frustrating and can lead adults to withdraw from a course (McAlister, 1998). In the flipped classroom outlined in this study, procedural and organizational autonomy allowed self-pacing to be a management skill, a sort of coping mechanism for falling behind. Remarkably, students who fell behind were able to catch up through the use of the video resources that were provided as part of the flipped classroom. Had these students not been able to access content delivery materials out of class time, they may have not been able to complete the course with so many absences, which could have led to withdrawal or failure.

Attendance

Some students downgraded to the self-paced mode of interaction after a series of absences because they had to catch up with the material that they fell behind with. Mark and Ryan encountered uncontrollable challenges in their lives that caused them to be absent due to external factors, altering their interactions in the class, while Lindsay chose to be absent due to internal factors related to her choice of interaction with the class. In particular, Mark missed several classes due to illness. After this period of absence, he began to work individually. At first, he used the self-paced mode of study to catch up with material he missed, but then he continued to use it in order to move ahead of schedule. Similarly, Ryan missed several classes due to a funeral, and then an English paper that took more precedence for him. After these periods of absence, Ryan used his class time in a self-paced manner in order to catch up with the material. He did not return to engaging in the complete class experience. Lindsay was also absent a lot. However, unlike the others, there was no significant external reason for her absence. She noted that she was absent when she was too far behind to participate in class activities. Absence seemed more like a coping mechanism for her.

Goals

Finally, the bifurcation into two key types of learning experience may be more prominently attributed to a variety in student goal orientations. Students who engaged in all components of the class (Alexa, Kristy, and Mark) tended to exhibit learning goal orientations. Whereas students who treated the class

in a self-paced manner (Ryan, Lindsay, and Vanessa) by opting out of the more collaborative class components, tended to portray performance goal orientations. Those with strong performance goal orientations evidenced a focus on efficiency in completing the course requirements. For example, Ryan agreed on the follow-up survey that he tended to avoid coming to class when he was behind because he "felt [he] could use [his] time more effectively outside of class, rather than covering more material [that he] would not understand." In contrast, Alexa and Kristy pursued learning activities regardless of whether they contributed to their grade of not.

Summary of analyses

Essentially, the students who engaged in the complete flipped classroom as presented in this paper were taking advantage of the collaborative elements of the class that provided them with cognitive autonomy support, primarily within the problem solving learning activities. These students held strong learning goal orientations. Meanwhile, the students who experienced the classroom in a self-paced manner focused on less collaborative components of the classroom where they could work individually and efficiently in an effort to satisfy their performance goal orientations. These students also tended towards embracing cognitive structure rather than cognitive autonomy support. The bifurcation of student experiences in the class occurred half way through the term when procedural and organizational autonomy support was more prominent. Students tended to opt-out of attending class or participating in group activities during this time if they were so inclined. It is interesting to note that once students downgraded to using the course in a self-paced manner, they did not return to using the elements of the course completely. However, what is most important is that all of the cases were able to successfully complete the course with a final grade of B+ or higher regardless of the manner in which they chose to experience the course. A visual summary of these analyses is provided in Table 2 below.

Table 2.
Summary of Analyses of Components by Case

	Alexa	Kristy	Mark	Ryan	Lindsay	Vanessa
Cognitive Autonomy Support	●	●	●	☾	☾	☾
Attendance	●	●	☾	☾	☾	●
Learning Goal Orientation	●	●	●	○	☾	☾
Performance Goal Orientation	☾	☾	○	●	●	●

● high   **Complete**   **Incomplete**
○ occasional   **Flipped Classroom**   **Self-Paced Classroom**
☾ low

Conclusions

The main intent of this research is to describe how students can experience a flipped classroom that is designed to promote flipped learning (The Flipped Network, 2014) and a transactional student-centred learning environment (Elen et al., 2007). Although the flipped classroom in this study afforded the capacity for a collaborative student-centred learning environment where the teacher was guided by student learning needs, it also provided students with an autonomous opportunity to choose ways in which they could interact in the class. In summary, students in the adult mathematics upgrading course Math 084 bifurcated into experiencing the flipped classroom in one of two ways: the complete flipped classroom and the self-paced option that the flipped classroom afforded.

Students who experienced the complete flipped classroom tended to exhibit strong learning goal orientations and engaged themselves autonomously in the collaborative learning tasks provided, the facilitative role of the teacher, and the social culture of learning in the classroom community. These students had more consistent class attendance and were less swayed by changes in organizational and procedural structure than their self-paced counterparts. On the contrary, students who experienced the flipped classroom as more of a self-paced classroom tended to exhibit strong performance goal orientations, often resisting cognitive autonomy support in an effort to maintain efficiency in completing tasks, and did not embrace engagement in collaborative learning opportunities.

Interestingly, this bifurcation of student interaction coincided with the increase in provision of procedural and organizational autonomy support in the latter half of the term during which class times were less structured procedurally and organizationally. As more organizational and procedural autonomy support was provided, self-paced performance oriented students tended to focus on completing the minimum requirements of the course. The bifurcation of student experiences also coincided with increasing student absences. Once students fell behind in the material or experienced a series of absences, they typically resorted to treating the course in a self-paced manner, an interaction that they continued until the completion of the term. It should be noted, however, that these students may have easily dropped out of the course had they not been provided with an extensive set of resources to help them complete the course as many adult students do when they fall behind in course material (McAllister, 1998).

Both the complete flipped classroom and the self-paced option that the flipped classroom afforded were highly student-centred and allowed students to pursue their goals orientations in the context of the course, regardless of their nature. Some students evidenced a shift in goal orientation from performance oriented to learning oriented, likely due to the contagious nature of engagement during collaborative problem based activities, but others did not exhibit this shift. Hence, it is important to note that although it is desirable for students to pursue goals of learning, it is not always what they desire. This speaks to the ever-present tension between student and teacher goals. It is also a good reminder of the fact that a goal cannot be forced onto anyone. Instead, the goal can be encouraged and nurtured through providing opportunities for developing deeper understanding if a student so desires. This is the essence of a student-centred learning environment.

The flipped classroom in this study provided students with an invitation to pursue goals of learning without forcing it to be the only option. Students could still complete the course and satisfy the prerequisites they needed by interacting in a self-paced manner, but more importantly, those who became interested in developing deeper meaning in mathematics were given the opportunity to do so through the collaborative nature of the classroom learning environment. Cognitive autonomy support in particular served as a determining factor in classroom interaction. This research supports the premise of Jang et al.'s (2010) theory that classrooms conducive to engagement give both structure and autonomy. In particular, organizational and procedural dimensions should be structured, while the cognitive dimension should be provided with autonomy support in order to promote student engagement in opportunities for collaboration with peers during meaningful classroom activities. Therefore, the main result of this research is that it affirms that cognitive autonomy support is an essential ingredient in promoting student engagement in learning opportunities.

Implications

The most important implication of this research for the adult mathematics education community is that it is an illustration of a learning environment that is conducive to providing adult students opportunities for pursuing goals of learning while maintaining accessibility of prerequisite completion through self-paced options. Although this study was conducted as a small scale exploration of six case studies in one particular implementation of the flipped classroom, it provides a basis for future research opportunities.

Future studies may want to look at exactly how each of the two ways of interaction in a flipped classroom (complete and self-paced) affect student understanding of the material in comparison to each other and in comparison to a control group that is not taught according to a flipped classroom model. Student achievement in a flipped classroom could also be studied further. All participants in this study who completed the course did so with a B- or higher. This leaves room for investigation of whether the flipped classroom in general pushes students into either succeeding in the course or dropping out of the course, or if it was just an instance that occurred within this small scale study.

Finally, a flipped classroom is merely a mindset with no clear method of implementation. Further implementation approaches could be explored. For example, content delivery videos could be used as content review rather than content preview. Class time could be treated in a more structured manner. Assessment strategies such as standards based grading could be also be explored. There are many opportunities for exploration of various approaches to flipped classroom implementation. That is the

beauty of the flipped classroom model: it provides a mode by which teachers can accomplish their goals of evolving a student-centred learning environment without compromising the delivery of the curriculum.

Acknowledgments

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What Do We Know about Mathematics Teaching and Learning of Multilingual Adults and Why Does it Matter? *

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Abstract

The significant role of language in mathematics teaching and learning is not a new phenomenon. Given the growth of cultural and economic migration, the increasing international focus on education for economic development and the widespread use of English as a language for learning, we have become acutely aware of the importance of language in adults' mathematics learning. While investigation has been undertaken in relation to the role of language in the learning and teaching of mathematics at primary and second level, little research has been done on multilingual (including bilingual) adults' learning of mathematics and the ways in which teaching might support such learning. In this paper we investigate the role of language in the mathematics and numeracy education of bi/multilingual adults with a focus on the mathematics register and discourse; we address the relationship between language(s) and learning; we provide a review of available literature specific to adult learners; and discuss implications for adult mathematics education.

Keywords: language, adult mathematics education, numeracy, bilingual/multilingual learners

Introduction

This paper is the latest in a series of papers for which the impetus was an international comparative study of adult numeracy education, focusing initially on the UK and New Zealand. That endeavour has so far resulted in a series of papers presented at successive international conferences of Adults Learning Mathematics – A Research Forum (ALM), starting at ALM17 in Oslo, Norway. At ALM20 in Newport, South Wales, we investigated language policy and adult numeracy education in Wales and New Zealand, focusing on the Māori language in New Zealand and the Welsh language in Wales (Coben & Miller-Reilly, 2014). In our ALM20 paper we noted that while much has been written about the relationship between language and literacy, the relationship between language and numeracy - especially adult numeracy - has been less explored, in particular from a policy perspective, despite evidence of the importance of language for learning. Accordingly, in that paper we sought to shed light on the policy context in which adult numeracy education is set in Wales and New Zealand with respect to those languages, viewed from a critical linguistic human rights perspective.

The need for further examination of the specific role of language in adult mathematics education is thus an important consideration emerging from our previous research. Language and communication are essential elements of teaching and learning mathematics, as is evident from research carried out in bi/multilingual settings (Gorgorió & Planas, 2001). Language facilitates the transmission of (mathematical) knowledge, values and beliefs, as well as cultural practices. Language is also the channel of communication within a mathematics classroom as language provides the tool for teacher-student and student-student interaction. Accordingly, we decided to examine the role of language in teaching and learning mathematics with and to adults. In this paper we are collaborating with Máire Ní Ríordáin, who has explored the role of language in mathematics learning with respect to Gaeilge, the Irish language (Ní Ríordáin, 2013; Ní Ríordáin & O'Donoghue, 2007, 2008; Ní Ríordáin & O'Donoghue, 2009, 2011). Our theoretical investigation focuses on the following main question: What is the role of language in the mathematics and numeracy education of bi/multilingual adults? This main question is broken down into the following sub-questions which are addressed in turn in the following sections:

- What can we learn from research on language and multilingualism?

- How should we understand the relationship between language and mathematics?

- What is meant by the 'mathematics register' and why is it important?

- What factors impede and promote mathematics learning in an additional language?

- What do we know about mathematics/numeracy education, language and adult learning?

- What are the implications for adult mathematics education in bi/multilingual settings?

Before answering these questions we wish to make clear that we recognise that there are differences between bilingualism and multilingualism, with their respective consequences for mathematics teaching and learning, and we use both terms in this paper as well as the composite term 'bi/ multilingualism' when we mean to indicate that these can be considered together. Similarly, terms such as ESL (English as a Second Language), ELF (English as a Lingua Franca), EAL (English as an Additional Language) and English as a Second or Other Language (ESOL) are used throughout this paper. We are aware that there are differences between these contexts for bilingual and multilingual learners and that more than one language may be involved (e.g., Breton as well as French in France, multiple languages in South Africa). However, this paper is presenting a theoretical perspective and is drawing on available literature published in English viewed through the lens of adult mathematics and numeracy education. We aim to be as comprehensive as possible within the scope of this paper because we want to contribute to opening up this important area for further research and development. We believe that to limit our investigation to either bilingualism or multilingualism, or to one context over another, might result in not presenting appropriate available research to help guide future research developments.

What can we learn from research on language and multilingualism?

There is growing recognition that language (and bilingualism/multilingualism) plays a key role in mathematics teaching and learning. Given the increase in international migration, the changing status of minority/indigenous groups and the dominance of English as a language for learning and teaching mathematics, many students face a transition to learning mathematics through the medium of another language (Barwell, Barton & Setati, 2007). Much diverse research has been undertaken on the effect of bilingualism/multilingualism on mathematics education, but it is beyond the scope of this paper to address all aspects. However, research on language and multilingualism from a range of perspectives has been reviewed by Canagarajah and Wurr (2011). They highlight the burgeoning research on English as a Lingua Franca (ELF) in which ELF is a locally achieved practice adopting negotiation strategies to achieve intelligibility and constructing intersubjective norms that are sufficient to achieve their communicative objectives. They point to an emerging synthesis in the research literature which treats:

- Competence as an adaptive response of finding equilibrium between one's resources and the factors in the context (participants, objectives, situational details), rather than a cognitive mastery of rational control;
- Cognition as working in context, in situ, distributed across diverse participants and social actors;
- Proficiency as not applying mental rules to situations, but aligning one's resources to situational demands, and shaping the environment to match the language resources one brings.

(Canagarajah & Wurr, 2011, p. 11)

We take this synthesis as the starting point of our exploration of language and multilingualism in relation to adult mathematics education. It chimes with our professional and academic experience in that it situates the language user (in our case, the adult learner of mathematics) as actively seeking to adapt and align their language use with the demands of the context. We draw on Grosjean's (1999) concept of bilingualism as a *continuum of modes,* with bilingual individuals using their languages independently and together depending on the context and purpose. We think this may shed light on some of the factors that promote and impede the mathematics learning of bilingual and multilingual adults. In particular, we are concerned with how bi/multilingual adult learners may use their languages when engaged in mathematical discourse and the process of learning, and how these language(s) may provide a set of linguistic resources for social and cognitive purposes, as described by Zahner and Moschkovich (2011). If competence is "an adaptive response of finding equilibrium between one's resources and the factors in the context (participants, objectives, situational details)" as Canagarajah and Wurr (2011, p. 11) propose, then adult mathematics and numeracy educators need to accommodate this in their teaching and learners need to know this is an effective strategy, albeit with some pitfalls for the unwary, rather than 'wrong'. Accordingly, it is of importance to examine the role of language in teaching and learning mathematics.

How should we understand the relationship between language and mathematics?

Given the significant role of language for the teaching and learning of mathematics, the language through which we initially learn mathematics will provide the mathematical foundations to be built upon and developed within that language (Gorgorió & Planas, 2001). A significant body of research exists supporting the view that different linguistic characteristics may impact on cognitive processing. Influential psychologists and educationalists, including Vygotsky and Bruner, have investigated the nature and relationship between language and thought. The primary concern to emerge from this research is whether language follows thought, thus making language a means for expressing our thoughts, or whether language determines and is a prerequisite for our thoughts (Brodie, 1989). The relationship between language and thought is extremely complex and conflicting views exist in the literature. However, the general consensus in cognitive science is to presume that thinking is occurring in some language (Sierpinska, 1994). Vygotsky was one of the earliest theorists to begin researching the area of learning and its association with language. He concluded that language is inextricably linked with thought – "the concept does not attain to individual and independent life until it had found a distinct linguistic embodiment" (Vygotsky, 1962, p. 4). Although a thought comes to life in external speech, in inner speech energy is focused on words to facilitate the generation of a thought. If this is the case, it raises an important question – does the nature of the language used affect the nature of the thought processes themselves? The transition from thought to language is complex as thought has its own structure. It is not an automatic process and thought only comes into being through meaning and fulfils itself in words. Thought is mediated both externally by signs and internally by word meanings (Vygotsky, 1962). Bruner (1975) emphasises that it is the use of language as an instrument of thinking that is of importance, as well as its affect on cognitive processing. Therefore, thought is intimately linked with language and ultimately conforms to it.

Mathematical language is considered as a distinct 'register' within a natural language. Therefore, the mathematics register in Irish will be different from the mathematics register in English, with each language possessing distinct ways and structures for expressing mathematical meaning and concepts. Of concern here is what the consequences of differences in languages are for adult mathematics

learners. For example, do the (mathematical) thinking processes of those learning mathematics through the medium of Irish differ from those learning mathematics through the medium of English? How are languages utilised in and impact on developments in mathematical discourses in adult education? The concept of the structure of a language impacting on thought processes is referred to as the linguistic-relativity hypothesis (Sapir, 1949; Whorf, 1956). The basic premise of this hypothesis is that the vocabulary and phraseology of a particular language influences the thinking and perception of speakers of this language, and that conceptions not encoded in their language will not be available to them. Hence, they are proposing that each language will have a different cognitive system and that this cognitive system will influence the speaker's perception of concepts (Whorf, 1956). Therefore, in theory, an Irish speaker/learner should have a different cognitive system to that of an English speaker/learner, influence our actions and accordingly may influence mathematical understanding. For example, Miura et al. (1994, p. 410) contend that 'numerical language characteristics *(East-Asian languages)* may have a significant effect on cognitive representation of number'. However, other researchers have questioned argued for the difficulty in applying the linguistic-relativity hypothesis and the difficulty in testing such claims in relation to mathematical thinking (Towse & Saxton, 1997). We acknowledge that this may be too strong of a way of viewing the influence of language on the mathematical thinking and less severe forms of this hypothesis have been proposed. We support the premise that language may not shape and determine our entire mathematical thinking, but that it may influence it to a certain degree and facilitates our thinking and perception (Sternberg & Sternberg, 2012). It follows from interpretation of this theory that the language through which we speak/learn facilitates our thinking and perception. When working with bilingual/multilingual learners, we need to be acutely aware of their languages and how these languages may impact on their mathematical thinking and learning as language is necessary to facilitate mental representation and manipulation of written mathematical text (Sierpinska, 1994).

'Understanding can be thought of as an actual or a potential mental experience' (Sierpinska, 1994, p. 1). Sierpinska defines these mental experiences as 'acts of understanding' as distinct from 'an understanding', which is the potential to experience an act of understanding. These acts of understanding occur at a particular time and are short in duration. In education, understanding is often correlated with cognitive activity over a longer period of time. In this 'process of understanding', 'acts of understanding' represent the important steps while the attained 'understandings' represent the supports for further development (Sierpinska, 1994). For many, understanding is often associated with meaning and/or understanding why (e.g., Piaget, 1978). Understanding can be described in relation to meaning, while meaning can be described in terms of understanding, thus heightening the confusion surrounding the topic. In order to be consistent in explaining the association between meaning and understanding, we consider that 'the object of understanding is the same as the object of meaning: it is the sign broadly understood' (Sierpinska, 1994, p. 23). Therefore, the concept/thought forms the basis of our understanding, while what we seek to understand are the signs that embody these concepts/thoughts. Because language and thought are interrelated (Bruner, 1975; Vygotsky, 1962) and thought is engaged in our understanding, then language is involved in developing our mathematical understanding. Understanding unveils a meaning: learners move from what the text states to grasping what the text is articulating (Sierpinska, 1994).

In 1979 Cummins refined his Threshold Hypothesis and this led to the development of his Developmental Interdependence Hypothesis, which had a more in-depth focus on the relationship between a student's two (or more) languages. The Interdependence Hypothesis proposed that the level of proficiency and use already achieved by a student in their first language would have an influence on the development of the student's proficiency and use of their second language. Cummins (1980) also addresses the importance of recognising that both languages interact and are stored together internally (Common Underlying Proficiency). Therefore, the impact of the first (and second, third, etc.) language of learning for mathematics is significant and needs examination when investigating bilingual/multilingual students. In particular, investigation is needed into how a particular language and its syntactical structure may impact on mathematical activity and reasoning (Morgan, Tang & Sfard, 2011). Galligan's (2001) extensive literature review in relation to differences between English and Chinese is the most significant review to be undertaken in relation to mathematics. She found that considerable differences exist in orthography, syntax, semantics and phonetics between the Chinese

and English languages and that these differences may impact on the processing of mathematical text. However, few other studies specifically exist in relation to a comparison of languages and impact on mathematics learning (Cai, 1998). In particular, there is a dearth of quality research in the relationship of adult numeracy teaching and learning and language, and it is to this issue that we turn next.

Moreover, we see mathematics as a discourse and a type of communication (Sfard, 2012). Discourse is more than just language. Our usage is close to that of Gee's Discourse, which he distinguishes with an upper-case D, while discourse (with a lower-case d) refers to language-in-use (Gee, 1990). As defined by Gee (1996, p. 131):

> A Discourse is a socially accepted association among ways of using language, other symbolic expressions, and 'artifacts,' of thinking, feeling, believing, valuing and acting that can be used to identify oneself as a member of a socially meaningful group or 'social network,' or to signal (that one is playing) a socially meaningful role.

By employing this definition, Discourses are more that verbal and written language and the use of technical language; Discourses also involve communities, points of view, beliefs and values, and pieces of work. Moschkovich (2012, p. 95) utilises the phrase 'mathematics Discourse practices' to draw attention to the fact that Discourses are embedded in sociocultural practices. The following description is provided:

> On the one hand, mathematical Discourse practices are social, cultural and discursive because they arise from communities and mark membership in different Discourse communities. On the other hand, they are also cognitive, because they involve thinking, signs, tools and meanings. Mathematical Discourses are embedded in sociocultural practices. Words, utterances or texts have different meanings, functions and goals depending on the practices in which they are embedded. Mathematical Discourses occur in the context of practices and practices are tied to communities. Mathematical Discourse practices are constituted by actions, meanings for utterances, foci of attention and goals: these actions, meanings, foci and goals are embedded in practices.'

(Moschkovich, 2012, p. 95)

Therefore, mathematical Discourse practices involve multi-semiotic systems (e.g. speech, writing, gestures, images, etc.) and thus are of importance when analysing mathematical teaching and learning in relation to bi/multilingual adults. Accordingly, we stress the importance of other factors such as exposure to mathematics, teaching strategies employed and culture as influencing attainment in mathematics, not just language (Towse & Saxton, 1997).

What is meant by the 'mathematics register' and why is it important?

We consider mathematical language as a distinct 'register' within a natural language, e.g., Gaeilge or English or French, which is described as "a set of meanings that is appropriate to a particular function of language, together with the words and structures which express these meanings" (Halliday, 1975, p. 65). One aspect of the mathematics register consists of the special vocabulary used in mathematics (Gibbs & Orton, 1994); it is the language specific to a particular situation type (Lemke, 1989). However, the mathematics register is more than just vocabulary and technical terms. It also contains words, phrases and methods of arguing within a given situation, conveyed through the use of natural language (Pimm, 1987). The grammar and vocabulary of the specialist language are not a matter of style but rather methods for expressing very diverse things (Ellerton & Wallace, 2004). Therefore, each language will have its own distinct mathematics register, encompassing ways in which mathematical meaning is expressed in that language. As is evident, the complex 'register' of mathematics is similar to a language *per se* and requires learning skills similar to those used in learning a language. This adds another dimension to mathematics learning and reinforces the view that the content of mathematics is not taught without language. The process of learning mathematics inevitably involves the mastery of the mathematics register (Setati, 2005). This allows students to communicate their mathematical findings in a suitable manner; "without this fluency, students are restricted in the ways that they can develop or redefine their mathematical understandings" (Meaney, 2005, p. 129). Developing a learner's mathematical register provides them with analytical, descriptive and problem solving skills within a language and a structure through which they can explain a wide

range of experiences. Once the register is mastered, learners will have the ability to listen, question and discuss, together with an ability to read, record and participate in mathematics. However, when working with adult bilingual/multilingual learners we need to be cognisant of each language having its own distinct mathematics register and ways of communicating mathematics. Similarly, registers exist in many disciplines (e.g., science, technology) but likewise ordinary/everyday English language can be classified as a register. The mathematics and ordinary language registers can interfere, often in subtle ways, in a learning environment. Thus learners need to recognise each of these registers so as to identify which is being used at any given time (Sierpinska, 1994) and this is a challenge many bi/multilingual learners encounter. We need to be aware of how different languages and registers, and use of multiple languages and registers, may help support the development of mathematical knowledge (Ní Ríordáin, 2013).

What factors impede and promote mathematics learning in an additional language?

It follows from the foregoing discussion of the mathematics register that mathematics is far from 'language free' and its particular vocabulary, syntax and discourse can cause problems for students learning it in a second language (Barton & Neville-Barton, 2003). Similarly, the concepts presented here are not limited to second language learners – these language features can impact on monolingual learners of mathematics. However, while many students who learn mathematics in their mother-tongue (e.g. Gaeilge) have difficulty in acquiring the mathematics register, this is heightened for those who must learn it in a second language (e.g., English). Learners have to cope with the new mathematics register, as well as the new language in which the mathematics is being taught (Setati & Adler, 2000). Some of the language features that may impede mathematical learning are discussed in the following sections. We propose that knowledge of such language factors can be utilised by teachers of bilingual/multilingual adult mathematics learners in order to promote and support mathematics learning.

A key issue that causes significant problems for second language learners (as well as monolingual learners) is the number of 'borrowed' words from everyday English (Pimm, 1987). These words tend to be ambiguous due to having one meaning in the mathematics register, while having another meaning in everyday use (Yushau & Bokhari, 2005). Examples of such words include average, degree, even, odd, operation, etc. The non-mathematical meanings of these terms can influence mathematical understanding, as well as being a source of confusion. Also, Rudner (1978) found that language features such as conditions (if, when); comparatives (greater than, the most); negatives (not, without); inferential (should, could); low information pronouns (it, something) can be sources of difficulty and hinder students' interpretation and understanding of mathematical word problems. Similarly, the use of specialist terms can lead to misinterpretation of mathematical tasks. Students tend to only encounter these terms within the mathematics classroom (for example, 'quadrilateral', 'parallelogram' and 'hypotenuse') and they are unlikely to be reinforced outside of it (Pimm, 1987). If second language learners do not acquire their correct meaning then this can lead to difficulties within the mathematics context. Second language learners have a tendency to translate new mathematical terms/vocabulary into their mother-tongue. There may be no equivalent translation and/or the translation may be done incorrectly, thus resulting in further confusion and misinterpretation (Graham, 1988). However, what is also worth considering is that specialist terms in some languages actually facilitate access to meaning and accordingly could be incorporated into teaching and supporting mathematics development for bilingual/multilingual adult learners. Such examples include the development of terms in Māori (Barton, Fairhall & Trinick, 1998), Chinese (Galligan, 2001) and terminology in the Irish language where, for example, velocity is 'treoluas' which when directly translated is 'direction speed' (Ní Ríordáin, 2013),

Context is also a key issue: 'Words can change their meaning depending on their context within the mathematics lesson' (Gibbs & Orton, 1994, p. 98). In terms of language analysis, this is known as semantics – establishing the meaning in language, or the relationship and representation between signs and symbols. Due to the multiple meanings that various words can have, the context is vital in determining the correct interpretation. This is very much connected to the use of specialist terms in

mathematics and multiple meanings of words (as discussed in the previous paragraph). This is by no means limited to the English language. For example, in Chinese context is essential as the language has relatively little grammar (Galligan, 2001). Findings from a review of literature found that children experience more difficulties with the semantic structure of word problems than with other contributing factors such as the vocabulary and symbolism of mathematics and standard arithmetic (Ellerton & Clarkson, 1996). Working within a bi/multilingual mathematics learning context would require attention being given to context and semantics and impact on mathematics learning. Finally, symbolism is one of the most distinctive features of mathematics, for example >, <, /, Σ. It is crucial for the construction and development of mathematics. Unfortunately "symbolism can accordingly cause considerable difficulties to those whose mother language has different structures" (Austin & Howson, 1979, p. 176). One of the requirements for mathematical learning is that students can interpret the mathematical text and convert it to an appropriate symbolic representation, and perform mathematical operations with these symbols (Brodie, 1989). Thus if students cannot understand the text (due to the language medium) they will be unable to convert it to the appropriate mathematical construction needed to solve the problem. Symbols provide structure, allow manipulation, and provide for reflection on the task completed.

What do we know about mathematics/numeracy education, language and adult learning?

While research has been done on language and mathematics learning, this has tended to focus on children's rather than adults' learning (e.g. Barwell, Leung, Morgan, & Street, 2002; Chval, Pinnow, & Thomas, 2015; Cuevas, 1984). A literature review by Benseman, Sutton and Lander (2005a) in New Zealand designed to provide a "critical evaluation" of the available research evidence on effective practices in literacy, numeracy and language (LNL) teaching and educational programme provision found "a dearth of specific research relating to this area in New Zealand and the situation is only marginally better overseas" and they were not able to identify any research that met the criteria for their review with regard to "factors associated with progress in numeracy" (Benseman et al., 2005a, p. 5). Earlier, Coben stated in the first major literature review of research in, and relevant to, adult numeracy:

> The research domain of adult numeracy is fast-developing but still under-researched and under-theorised. It may be understood in relation to mathematics education, as well as to adult literacy and language, and to lifelong learning generally.

> (Coben et al., 2003, p. 117)

Similarly, a review of the literature in adult numeracy conducted in the U.S.A. found, in common with the findings of earlier reviews (Coben et al., 2003; Tout & Schmitt, 2002), that "very few research studies have used Adult Basic Education students to study the effects of adult numeracy instruction, and the research that does exist is neither theory-driven nor guided by any systematic approach" (Condelli et al., 2006, p. 23).

In the years following these reviews, the National Research and Development Centre for Adult Literacy and Numeracy (NRDC) in England undertook a series of 'effective practice' studies, including a study of effective practice in adult numeracy (Coben et al., 2007). The summary of findings for that study highlighted the diversity of adult numeracy learners and education, including the fact that many learners were bi/multilingual:

> The heterogeneous nature of adult numeracy teaching, the range of learners and the number of variables amongst teachers and learners, make it difficult to identify effective practices and factors that can be generalised with confidence across the whole sector. [...] There were particular difficulties for adults with lower ability levels, and with reading or language difficulties (two in every five learners in the sample spoke English as an additional language).

> (Coben et al., 2007, p. 10)

Also, over the period from 1998 to 2007, the Longitudinal Study of Adult Learning (LSAL) investigated the development of 'literacy' (subsuming 'numeracy' into 'literacy') in adult life of a

target population of about 1000 high school dropouts. At the start of the study, participants were aged 18-44, proficient but not necessarily native English speakers, and residents of Portland, Oregon, USA. Perhaps because the participants were all proficient English speakers, language issues do not feature in the findings, that: literacy development varies and continues to develop in adult life after leaving school: age has an effect on literacy growth; literacy measures are correlated; life history events have effects on literacy development; participation in programmes and self-study have patterns of effect on literacy development; and there are strong effects of programme participation on adults' subsequent perceptions of improved literacy (Reder, 2012). Also, long-term effects on proficiency bear out the predictions of practice engagement theory (Reder, 1994; Sheehan-Holt & Smith, 2000) in that engagement in literacy practices leads to growth in literacy proficiency and literacy development in adulthood affects employment and earnings (Reder, 2012). The NRDC 'Effective Practice in Adult Numeracy' study and LSAL have to an extent filled the gap identified by Coben et al. in 2003 and Benseman et al. in 2005a, but still much remains to be done before we can say that understanding of instructional impacts on adults learning numeracy/mathematics *and language* is sufficiently strong to support clear guidance on teaching and learning.

Research is also limited on the impact of workplace numeracy programmes, and on workplace ESOL (English for Speakers of Other Languages) and ESL (English as a Second Language) programmes. For example, Barker writes with regard to workplace literacy: "Empirical studies on the impact of workplace literacy programmes are not common, indeed the whole area of evaluation of training is underdeveloped" (Barker, 2001, p. 28). Gray (2006, p. 45) states that "information on outcomes from workplace ESL instruction is also lacking. The research studies that do exist are generally case studies or qualitative research." A review by the American Institutes for Research (AIR) which focused on ESL students in Adult Basic Education also found very few studies with Adult Basic Education and ESL students which demonstrated a statistically significant impact of instruction (Condelli, Wrigley & Yoon, 2008). As was the case also for Benseman, Sutton and Lander's literature review (Benseman et al., 2005a), the scope of the criteria used to select studies was extended. Condelli, Wrigley and Koon found that over the last 30 years there had been extensive international research into second language, as distinct from (first language) literacy, teaching and learning, but this was not the case in the numeracy field. A search for research about numeracy for ESL students by an AIR team in 2006 revealed none: "There exists no research base at all on how numeracy is taught in ESL classes, let alone studies that examine instructional approaches and their impact on these learners" (Condelli et al., 2006, p. 34).

In the first major review of adult numeracy only a few studies were found that addressed research on numeracy and ESL students and it was concluded that "Very little is known about learning and teaching adult numeracy with adults who speak languages other than English; research is needed in this area" (Coben et al., 2003, p. 118). This conclusion is echoed by AIR researchers, who suggested five areas for further research in order to move the field of Adult Basic Education forward. The fifth area suggested by the AIR researchers for further research is relevant to this paper, namely, to "examine (instruction for) ESL learners and students with learning disabilities" since these areas are "completely neglected" (Condelli et al., 2006, p. 61). The review goes on to state:

> We found no research on how to provide instruction to these learners, on how they learn, or on how to address the challenges these learners face in learning mathematics. Research needs to pay particular attention to instruction for adult ESL learners, who make up over 40 percent of students in adult literacy programmes.

> (Condelli et al., 2006, p. 62)

Benseman, Sutton and Lander (Benseman et al., 2005a) found that various factors appear likely to enhance learner gain in literacy, numeracy and language. These include appropriately skilled teachers able to identify learners' strengths and weaknesses in speaking, reading, writing and numeracy and undertaking deliberate, explicit and sustained acts of teaching, clearly focused on learners' diagnosed needs, using a curriculum linked to learners' lived experience. Programmes of over 100 hours of tuition are recommended, allowing for high levels of participation, using a range of clearly structured teaching methods. They recommend ongoing assessment, taking account of the variation in learners'

skills and a focus on efforts to retain learners, including pro-active management of the positive and negative forces that help and hinder persistence as well as family-focused programmes with a clear focus on literacy and numeracy development. They also contend that ESOL programmes may need to be longer and should be structured to maximise oral communication, discussion and group work based on real-world situations, texts and tasks. Computers and multi-media technology can provide useful support, with bi-lingual teaching to explain concepts and learning tasks.

The authors note that an "emphasis on individualized teaching and learning may not support the needs of adult ESOL learners" for talking and interaction and such an individualised approach may be more common in adult numeracy classrooms than in other areas of education (Benseman et al., 2005a, p. 78). In addition, they state that: "the use of everyday, culturally-specific situations to contextualise maths problems may act as a barrier to attainment for ESOL learners in numeracy classes, when they don't have either sufficient language knowledge or contextual experience" (Benseman et al., 2005a, p. 78). Such cautionary notes are necessary, since, as Ciancone (1996) acknowledges, some educators working with adult learners may not be experienced with the teaching and learning of mathematics in a contextualised manner and linked with literacy. Marr's (2000) work demonstrates "that aspects of language acquisition will develop when supplemented with conceptual tasks and activities that focus on the written and oral use of mathematical understandings" (Coben et al., 2003, p. 113). Numeracy taught in this way might benefit all adult students as they learn the mathematics register, and might particularly help students learning English as an additional language in the numeracy classroom. A kit designed to develop the adult numeracy skills of literacy and mathematics trained teachers in Australia using a participatory workshop approach includes a chapter on 'Language and Maths' which includes theoretical and background information and provides guidelines for integrating literacy and numeracy with adult learners (but not ESOL learners) in the classroom (Marr & Helme, 1991).

In particular, studies focused on the teaching of mathematics/numeracy to multilingual adult learners are limited. The aim of an observational study of 15 literacy, numeracy and language teachers (Benseman, Sutton & Lander, 2005b, p. 92) was to gain an overview of how teachers teach literacy, numeracy and language in New Zealand. In the event, little numeracy teaching was observed in this study. These teachers came from tertiary institutions, community organisations, workplaces and private training establishments. The authors of the review comment on the diversity of the sector and the challenges of teaching adults literacy, numeracy and language. They conclude that teachers' commitment, empathy and support for their learners stem from the teachers' strong belief in the value of their programmes and their intrinsic interest in their work. The range of teaching methods observed was limited (both generic and literacy, numeracy and language-related). The authors note that generic teaching and classroom management skills play a significant role in literacy, numeracy and language teaching and the teachers appeared to use the same teaching strategies for ESOL as for others for whom English was a first language.

Ciancone gives guidelines for teaching numeracy in an ESL literacy programme, drawing on a range of sources (Ciancone & Jay, 1991; Kallenbach, 1994; Leonelli & Schwendeman, 1994; Lucas, Dondertman & Ciancone, 1991). He recommends: encouraging learners to look for patterns rather than just finding the right answer; pointing out to them that there may be many ways to solve the same problem; encouraging peer-group collaboration: he argues that the best way to clarify one's own understanding of a concept is to explain it to someone else. He also recommends encouraging learners to write journals about their mathematics learning and their feelings about learning mathematics, because using the language of mathematics reinforces both the mathematical concepts taught and the learner's proficiency in English. He notes that although numeracy is an everyday coping skill, mathematical concepts can be quite abstract and advocates using more concrete and visual explanations to facilitate understanding of the abstract concept. He advocates that each numeracy lesson should provide a balance between skill building and functional needs. For example, a lesson might begin with a problem (e.g. a mistake on a pay check) that provides a context for learning new skills (such as subtracting decimals), or it might start with a skill (e.g. adding decimals) followed by practical applications (such as adding sales tax to a fast food bill). He argues that mathematics should be included in literacy instruction from the beginning and that even learners who have almost no proficiency in English need to learn numbers for such basic activities as shopping and riding the bus (Ciancone, 1996, p. 4).

Finally, consideration must be given to EAL adults in post-compulsory mathematics education (e.g. undergraduate). Barton, Chan, King, Neville-Barton and Sneddon (2005), investigating the issue surrounding the learning of mathematics at university by students who have English as an Additional Language (EAL students), showed that the problems experienced by these students are not experienced by students whose first language is English (L1 students). EAL students struggle with their learning of mathematics in English at undergraduate level much more than has been appreciated. The effect is masked at Year 1 undergraduate level because of the better mathematical prior knowledge of EAL students and the relatively low language requirements at this level. However, the effect in the third year is much greater. This study of third-year undergraduate students confirms for the first time that specific features of mathematical discourse cause difficulty for EAL students. Discourse density and logical structure are particularly confirmed as critical in this study, although comprehending mathematical discourse as a whole is also found to be much more complex than anticipated. In addition, these EAL students are unaware of their difficulties. These authors suggest that Departments of Mathematics need to acknowledge and address this issue in realistic ways.

What are the implications for adult mathematics education in bi/multilingual settings?

As we have seen, the literature specifically dealing with *adult* mathematics education in bi/multilingual settings is sparse. The considerably greater body of research on children's mathematics learning in relation to language, and on language learning more generally, may give some indications of ways forward. For example, Winslow concludes that

> The view of mathematics as characterized by a specific linguistic register, including an analysis of the nature and functions of this *mathematical register*, enabled me to provide several arguments for mathematics teaching in an ambitious sense which emphasises communicative abilities developed thereby, and to formulate the arguments in a way which indicate the position of mathematics among other forms of communication in which human knowledge (and school subjects) appear. In particular, I have stressed the importance of general fluency in the mathematical register for global, noise-free human interaction, for participation in human society and for popular engagement in the dialogue between humanity and nature. I have pointed out the contours of possible need for a complete revision of mathematics teaching, both in the choice of emphasis and in its temporal placement within the educational system.

> (Winslow, 1998, p. 23)

Winslow's 'linguistic approach to the justification problem in mathematics education' chimes with the findings of Schleppegrell's (2007) review of research, which suggests that focusing on the features of the language through which mathematics is constructed can be a strategy for engaging students and supporting their learning. Schleppegrell stresses that "focus on meaning, not form, is key to all discussion about mathematics concepts" (Schleppegrell, 2007, p. 151). However, she points out that exposure to the mathematics register through teacher's talk or textbook, or interaction with peers, will not in itself lead to learners developing the mathematics register. She cites research by Adams (2003) who suggests that teachers can help students to move from everyday language into the mathematics register by helping students recognize and use technical language rather than informal language when they are defining and explaining concepts; by working to develop connections between the everyday meanings of words and their mathematical meanings, especially for ambiguous terms, homonyms, and similar-sounding words; and by explicitly evaluating students' ability to use technical language appropriately. There is a tension between the formal register of academic mathematics and the 'everyday' or 'functional' focus of many adult numeracy programmes. Evidence-based ways forward have been proposed by various researchers. For example, contributors to an edited collection exploring the mathematics education of Latinos/as present research that grounds mathematics instruction with and for these learners in the resources to be found in culture and language. Language (e.g., bilingualism) is thus not framed as an obstacle to learning, but as a resource for mathematical reasoning and learning, in and out of school (Téllez, Moschkovich & Civil, 2011). Prediger, Clarkson and Bose (2012) also propose a way forward for teaching in multilingual contexts. They do this through purposefully relating multilingual registers. They explore the overlap between the three

different strong ideas related to different language registers and discourses: code switching, transitions between informal and academic (mathematical) forms of language within a given language, and transitions between different mathematical representations. They argue that integrating these ideas has the potential to enhance language-sensitive teaching strategies in multilingual classrooms that aim for conceptual understanding: an insight that might be fruitful for adult educators. Schleppegrell argues for more active engagement with the mathematics register and concludes that:

> We have seen that features of the mathematics register can be identified and analysed by students to see how meaning is made in mathematics. Teachers can support the development of the multi-semiotic mathematics register through oral language that moves from the everyday to the technical mode. Students can be encourage to produce extended discourse in mathematics classrooms, engage in discussion about the language through which word problems are constructed, and practice the writing of mathematics concepts in authentic ways. Teachers can become aware of the linguistic issues in learning and teaching mathematics and can develop tools for talking about language in ways that enable them to engage productively with students in constructing mathematics knowledge. Further research by applied linguists and mathematics educators can explore the linguistic challenges of mathematics learning in its multi-semiotic complexity to provide more support for teachers who want to engage struggling learners.
>
> (Schleppegrell, 2007, pp. 156-157)

We conclude that there is a clear need to develop specific recommendations in relation to multilingual adult learners of mathematics in order to address their specific needs and to facilitate participation in mathematical discourse.

Summary – why does it matter?

This paper explored aspects of practice concerned with mathematics teaching and learning in relation to multilingual adults. But why does this matter? The importance of language for the teaching, learning, understanding and communication of mathematics cannot be ignored. Educational objectives require students to understand mathematical concepts and to possess an ability to express their understanding of these concepts in written format (Gerber, Engelbrecht, Harding & Rogan, 2005). However, the function of language does not lie solely in the representation of mathematical knowledge. Language is required for and engaged in bringing this knowledge into existence (Halliday & Martin, 1993). Furthermore, mathematics learners are required to possess competency both in everyday language and mathematics-specific language, but competency in the natural language does not necessarily contribute to competency in the mathematics specific language (Lemke, 1989). Clearly the intricate relationship between mathematics learning and a student's language is highly complex. This is further complicated when the language of instruction/learning changes, as is the situation faced by many multilingual adult mathematics/numeracy learners. Moreover, we need to consider mathematics as a discourse (or Discourse, in Gee's terms) and one that is not singular or homogenous (Moschkovich, 2012). Accordingly, mathematical learners use multiple resources from their experiences (both in and outside of the learning context) and we need to be cognisant of multiple registers co-existing in the learning environment. Therefore, addressing the needs of multilingual adult learners is of paramount importance. Bi/multilingual learners should not be viewed in a deficit mode. Rather, their language(s) should be viewed as a resource for learning mathematics. However, as demonstrated in this research paper, this area is under-researched and under-theorised. Research practices/findings generated from participants from a dominant group (e.g. monolingual speakers) assumes these to be the norm for all adult learners. We endorse a call for more research in relation to multilingual adult learning, and we suggest that ALM, as an international and (to an extent) multilingual organization, is ideally placed to generate such studies and in so doing to increase understanding of the important and neglected area of the relationships between language and mathematics learning and teaching with and for adults. The need for more research and debate, as well as language-informed and language-friendly policy, is evident. Only then can we tackle these issues through pedagogic and support measures.

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The Hamilton Walk and Its Positive Impact on Adults Learning Mathematics Outside the Classroom

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Abstract

This paper discusses how the annual Hamilton walk has a positive impact on adults learning mathematics outside the classroom. The walk takes place on October 16 and typically attracts about 150 people from a wide variety of backgrounds, including many from the general public. I will show how the walk has changed many adults' perception of mathematics for the better and enhanced their understanding, appreciation and awareness of mathematics. Also, the walk has inspired many artists to create art works in relation to Hamilton, including poems, paintings, a song and more.

How it All Began

On October 16, 1843, Ireland's greatest mathematician, William Rowan Hamilton, was walking along the banks of the Royal Canal in Dublin when he had a Eureka moment. Hamilton's creation of a strange new system of four--dimensional numbers called quaternions on that day was a major moment in the history of mathematics and would change the world of mathematics forever. Hamilton later described his Eureka moment to his son in a letter, as follows:

> *Although your mother talked with me now and then, yet an undercurrent of thought was going on in my mind, which gave at last a result, whereof it is not too much to say that I felt at once an importance. An electric current seemed to close; and a spark flashed forth, the herald (as I foresaw, immediately) of many long years to come of definitely directed thought and work ... Nor could I resist the impulse -- unphilosophical as it may have been -to cut with a knife on a stone of Brougham Bridge as we passed it, the fundamental formula...*

The annual Hamilton walk takes place on October 16 and commemorates Hamilton's creation of quaternions by retracing his steps from Dunsink Observatory to Broombridge where he created quaternions. Before I discuss the walk in more detail, I will tell more of the story of Hamilton's Eureka moment. This story is also part of the beginning of the walk and the end of the walk (and potentially during the walk).

Hamilton's creation of quaternions was his most celebrated contribution to mathematics. Number couples (or complex numbers) were important in mathematics and science when working in two dimensional geometry and Hamilton had been trying to extend his theory of number couples to a theory of number triples (or triplets). Hamilton hoped the triplets would give a natural mathematical structure and a new way for describing the three dimensional world. He was finding it difficult to define the multiplication operation in his search for a suitable theory of triplets. We now know why he was having such difficulty because it's impossible to create the suitable theory of triplets.

Then, in a spark of inspiration, Hamilton's mind gave birth to quaternions on the banks of the Royal Canal on October 16, 1843. In an act of graffiti, he scratched his quaternion formulas on the bridge as described in his letter above. This piece of mathematical vandalism would change the world of mathematics forever. Hamilton reckoned that if he worked with number quadruples and an unusual multiplication, then he would obtain all that he desired. He called his new system of numbers Quaternions because every number quadruple contained four components.

The mathematical community was stunned at his audacity in creating a system of numbers that broke the usual commutative rule for multiplication (ab=ba). Hamilton was called the Liberator of Algebra since his quaternions smashed the previously accepted convention that a useful algebraic number system should satisfy the rules of ordinary arithmetic. Quaternions opened up a whole new mathematical landscape in which mathematicians were now free to conceive new algebraic systems which were not constrained by the rules of ordinary arithmetic. I suppose one could say that it was:

> *One small scratch for a man, one giant leap for mathematics!*

An important part of the Hamilton walk is to say, at the beginning of the walk, that Hamilton's motivation for doing mathematics was the quest for beauty in mathematics. I believe that any discussion of beauty in mathematics needs to start with the statement that mathematics essentially consists of an abundance of ideas. Number and circle are just some examples of the myriad ideas in mathematics. I find from experience in promoting mathematics with adults outside the classroom, that it can come as a surprise when they hear that number is an idea that cannot be sensed with our five physical senses. Yet, numbers are indispensable in society today and arise almost everywhere from football scores to the time of day.

The reason number arises practically everywhere is because number is an idea and not something physical. Many adults think that they physically see the number two on a blackboard but this is not so. The number two cannot be physically sensed because it's an idea. What appears on the blackboard is merely a symbol to represent the idea we call two.

Mathematical ideas like number can only be really 'seen' with the 'eyes of the mind' because that is how one 'sees' ideas. Think of the analogy with the sheet of music versus the music. The sheet of music is important and useful but it's nowhere near as interesting, beautiful or powerful as the music it represents. Similarly, mathematical notation and symbols on a blackboard are just like the sheet of music; they are important and useful but they are nowhere near as interesting, beautiful or powerful as the actual mathematics (ideas) they represent. When some adults outside the classroom first become aware that mathematics comprises ideas, it's as if they have just first become aware of the existence of music (that is not just a sheet of music). Consequently, this changes many adults' perception of mathematics for the better (in a major way).

Many adults say they do not see mathematics in the physical world and this is because they are looking with the wrong eyes. These adults are not looking with the eyes of their mind. As part of the walk, I ask adults to look with the eyes of their mind. They then begin to 'see' mathematics in places they never imagined.

As mentioned above, at the start of the walk I say that Hamilton's motivation for doing mathematics was the pursuit of beauty. Now, the beauty in mathematics typically lies in the beauty of ideas, because as stated earlier, mathematics essentially comprises an abundance of ideas. Our notion of beauty usually relates to our five senses, like a beautiful taste or a beautiful vision etc. The notion of beauty in relation to our five senses plays a very important part in our society. However, I believe that ideas (which may be unrelated to our five senses) can also have beauty and this is where you will typically find beauty in mathematics. Consequently, to experience beauty in mathematics, adults typically need to look, not with their physical eyes, but with the eyes of their mind because that is how one 'sees' ideas. I believe that beauty is the most important feature in mathematics (Ó Cairbre, 2009).

The notion of freedom in mathematics is also an important part of the walk because, as mentioned above, Hamilton was called the Liberator of Algebra. Hamilton was free to create a new system of

numbers that did not satisfy the rules of ordinary arithmetic, even though this broke the accepted convention in the mathematical community at the time. The notion of freedom in mathematics shocks many adults. However, as Cantor once said:

> *The essence of mathematics lies in its freedom.*

The reason freedom is an important feature of mathematics is because one is free to conceive of any new ideas one wants in mathematics. These new ideas may or may not lead to anything interesting or useful. Historically (and probably also in the future), the big breakthroughs in mathematics have typically occurred because the great mathematicians felt free to conceive of any new ideas they wanted even if their wild thoughts broke the conventions and seemed strange to other mathematicians and the general public. Hamilton's creation of quaternions is one such example. I believe there is great educational value in adults hearing about how freedom in mathematics has led to many crucial breakthroughs in mathematics.

The Annual Hamilton Walk

The annual Hamilton walk was initiated by Anthony G. O'Farrell in 1990. I organise it now and it typically attracts about 150 people from diverse backgrounds including staff and students from third level, second level and many from the general public. A Nobel Prize winner or a Fields Medallist often participates in the walk. The walk takes about forty five minutes and ends up at Broombridge in Cabra in Dublin, where a plaque on the bridge commemorates the creation of quaternions. I suppose one could call the annual walk a pilgribridge! The large number in attendance from the general public indicates a substantial public interest in the walk. Also, I get many calls from the media (television, radio and newspaper) and other bodies every year expressing an interest in doing a piece on Hamilton and the walk. Thus, the walk has appeared six times on television and an abundance of times on a variety of radio programmes and in many newspaper articles and I have given lots of talks on Hamilton. The walk has built up a large momentum of its own in the sense that it seems to be well known in the general public even outside mathematical circles. See Ó Cairbre, 2010 for a history of the walk. Anybody interested in coming on the walk should contact me.

The local Cabra Community Council make the annual walk into a very festive event with a large banner draped across the bridge and stalls along the canal. Broombridge is now a world famous location in the history of mathematics and science because of Hamilton's creation of quaternions. The word, Broomsday, is now often used in mathematical and public arenas to indicate October 16 and the walk. This word plays the same role as Bloomsday for literary groups and James Joyce fans. Actually, since I mentioned James Joyce, I may as well say that quaternions appear in Joyce's celebrated book, Finnegans Wake.

> *Wondering was it Hebrew set to himmeltones or the quicksilversong of qwaternions; his troubles may be over but his doubles have still to come.*

In this quote, Joyce is conflating Hamilton with two other Hamiltons. James Archibald Hamilton was the first astronomer at Armagh Observatory and he observed the transit of Mercury, i.e. quicksilver. James Hamilton was a clergyman who published a book on psalms.

Positive Impact of the Walk

The walk is a great example that relates to adults learning mathematics outside the classroom. It's also important to note that adults participate in the walk voluntarily and many of the adults on the walk are outside mainstream mathematics education. The walk brings to life a famous event in the history of mathematics and gives adults the opportunity to interact with Hamilton's story. In this way, adults feel much more connected to Hamilton's story than if they were just reading some mathematics in a book.

One of the main aspects of the positive impact of the walk on adults is that the walk changes many adults' perception of mathematics for the better. The walk can also lead to a very positive image of

mathematics in a whole community. Here are some examples of quotes related to the above comments:

On account of the walk, Hamilton is in the folk consciousness of the local people.

Local Cabra resident, Jack Gannon, who was also inspired by the walk to write a ballad about Hamilton in 2003.

The walk has had a huge impact on the local community. In fact it has gone way beyond just being a walk because all the local school children and the community are extremely proud of Hamilton and their local connection with him. The walk really has touched the local community in a big way. The fact that famous mathematicians and Nobel Prize winners mingle with school children and the local community on the walk and at the bridge is a great experience. Also, not one but two local artists have been commissioned in recent times to do portraits of Hamilton which are then publicly displayed at the bridge during the walk.

(Aodhán Perry of Cabra Community Council)

The Hamilton walk was my licence to explore and express myself around the subject of mathematics. By the age of nine I had decided I couldn't do mathematics, but I had also developed a strong interest in things technical and scientific and this created a conflict that simmered in the background of my educational and professional career for forty years. That is until I took part on the Hamilton walk in 2005. That walk had a profound effect on me. Hearing not only a Nobel laureate and a professor of mathematics sing Hamilton's praises, but also local poets, school children, balladeers and the Cabra community council, spurred me to turn my desire to celebrate Ireland's Science Heritage into action. That action turned out to be a family run business called Science Heritage Ireland.

(Mick Kelly from Swords in Co. Dublin)

Mick also wrote the following:

By the 2007 walk I could sense flaws developing in the glass wall I had built around learning mathematics and found it strange but very uplifting to be answering queries from people about quaternion algebra. There was a special sense of magic at Broombridge on that fine Tuesday, October 16, 2007, when the canal bank was alive with children playing all kinds of mathematics games. I couldn't help but wonder how many bridges to the future the organisers of this walk and maths week had created for our children.

I believe this change of perception can be crucial in any form of positive mathematics education, because after the change in perception, adults are then more likely to enhance their understanding, awareness and appreciation of mathematics. I see this happening frequently in relation to the Hamilton walk.

Many adults tell me that one of the main reasons their perception of mathematics changes for the better is because the walk introduces them to the fact that mathematics comprises an abundance of ideas and consequently beauty and freedom/creativity can be important features of mathematics. Another positive impact on adults is the storytelling approach to mathematics which happens naturally as part of the walk. Also, adults say that the sheet of music versus the music (analogy), mentioned earlier in this paper as part of the walk, has a big impact on changing their perception of mathematics for the better. When they realise, for the first time, that mathematics exists in the world of ideas (rather than the physical world), it's as if they become aware of music (rather than the sheet of music) for the first time. It really is that extreme in a positive sense.

I believe that this sheet of music versus the music (analogy) should be presented early when adults are learning mathematics. In other words, adults should be made aware early that mathematics comprises an abundance of ideas (rather than being just a load of symbols and notation), just like people are aware of music early (rather than music being just a load of notes on a sheet).

I have been promoting mathematics in the public arena for many years now. The annual Hamilton walk probably has the biggest impact on changing adults' perception of mathematics for the better. I believe this is because the walk (and the accompanying Hamilton story) has all, what I call, the "big picture of mathematics" features.

Here is a list of these "big picture of mathematics" features:

a) Storytelling

b) Drama

c) Humour

d) History of mathematics

e) Human element and famous characters

f) Beauty

g) Freedom and creativity/imagination

h) Practical power and applications

i) Motivation

j) Irish connections

k) Research and unsolved problems

l) Word origins

m) Outdoor activities

n) Cultural connections

o) Tricks/magic

p) Puzzles

a) Mystery

b) Wonder

c) Deductive reasoning

d) Abstraction

All of the above features can be relevant to adults learning mathematics outside the classroom. I will elaborate on some of the items on the list above. The drama related to the walk follows naturally from Hamilton's Eureka moment mentioned at the beginning of this paper. Beauty and freedom have

already been discussed in relation to the walk. Now I will mention a variety of examples of the practical power and applications of quaternions that can be mentioned at the beginning of the walk.

i. Quaternions play an important role in computer games. One example of this, which always appeals to journalists, radio hosts and students of course, is the fact that Lara Croft in Tombraider was created using quaternions!

ii. Continuing with the theme of entertainment, quaternions now play a significant role in computer animation and special effects in movies. For example, an Irish company called Havok used quaternions in the creation of the acclaimed new special effects in the film, The Matrix Reloaded, and also in the movie, Poseidon, which was nominated for an Oscar for its visual effects in 2007. Havok also received an Emmy award in the US in 2008 for pioneering new levels of realism and interactivity in films and games.

iii. Quaternions play a crucial part in space navigation. For example, they were fundamental in the Apollo 11 landing on the moon in 1969 and the Curiosity Rover landing on Mars in 2012.

iv. Quaternions played a part in Maxwell's mathematical theory and prediction of electromagnetic waves in 1864. This theory eventually led to the detection of radio waves by Hertz. Thus, the inventions of radio, television, radar and many other important products of our society are directly related to quaternions.

v. Vector analysis, which is indispensable in Physics, is an offspring of quaternions.

The walk and Hamilton's story has been an inspiration for a diversity of artists to create works of art related to Hamilton. Also, there has been a wide variety of public interactions with Hamilton's story and the walk. Here are some (of many) examples:

a) Sculptures (busts, bog oak and sand)

b) Paintings

c) Poems

d) Stamps

e) Plaques

f) Graffiti art

g) A song

h) A statue

i) A coin

j) A video installation in an art exhibition

k) Television news and documentaries

l) Radio shows

m) Newspaper articles

n) Housing areas

o) Institutes

p) T-shirts

q) A building

r) A moon crater

s) A golf hole

t) A banner

u) A bridge

v) References in James Joyce's Finnegans Wake, Flann O'Brien's work and a play by Sebastian Barry.

All of the above can be relevant to adults learning mathematics outside the classroom. Why all the inspiration? I believe the reason again is because all the "big picture of mathematics" features are present in the walk and Hamilton's story.

Now, I will discuss some of the examples above in more detail. The walk inspired Jack Gannon to write a song about Hamilton. This song has been played many times on radio and also on some television shows. Consequently, the song plays a role in adults learning mathematics outside the classroom. The walk also inspired the local community to commission two portrait paintings of Hamilton, which are publicly displayed at the bridge during the walk. Mick Kelly's T-shirts were also a consequence of his participation in the walk. The walk has appeared on television six times and on many radio shows and in lots of newspaper articles and thus the general media audience can hear about the walk in a variety of ways. The general public plays a significant role in mathematics education at all levels because parents, policy makers and the media are all members of the general public and can exert great influence on the attitude of young people and adults towards mathematics.

A current/future example of the impact of the walk on adults relates to Dublin City Council. In the last month I have been contacted by the City Council because they are doing an exhibition on Hamilton later this year. The exhibition will run from September to November and the main artist wishes to interact with me in relation to Hamilton. One example of this interaction will be to incorporate the walk into the exhibition.

There are a variety of places where one can read about Hamilton's life and works (Hankins, 1980; Ó Cairbre, 2000 & O'Donnell, 1983).

Conclusions

The annual Hamilton walk is a great example of something related to the history of mathematics that continues to inspire adults outside the classroom. Many of these adults are initially outside mathematics and then the walk changes their perception of mathematics for the better. Consequently, the walk has a positive impact on adults learning mathematics outside the classroom.

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If Self-Efficacy Deficiency Is the Disease,
What Treatments Provide Hope for a Cure?

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Abstract

Several studies have found that a sense of self-efficacy is the best predictor of success in the mathematics classroom for adult students. A lack thereof puts students at risk for failure. If this is true, then it would seem useful to identify research projects whose classroom methods strive to stem the tide of defeat and promote feelings of self-efficacy in students. The paper provides an introduction to the concept of self-efficacy and a summary of the dissertation research examining the impact of self-efficacy on adult mathematics students. It summarizes the findings from research reports that have evaluated the impact of interventions with adult students. Finally, it casts a wider net and examines self-efficacy projects from elementary and secondary school venues that might be useful to teachers in adult mathematics education.

Keywords: literature review, teaching, mathematics

Introduction

Two year ago, at ALM-19, I presented a paper that summarized research about adult mathematics education published in the United States since 2000 (Safford-Ramus and Rotondo, 2013). Many of the doctoral dissertations reviewed for that report had as their primary or secondary theme the interplay of student self-efficacy and success or failure in their studies. Repeatedly, self-efficacy served as the best predictor of a favorable outcome. Findings from my own doctoral research had hinted at this fact, but it had not been the subject of my research questions.

In my courses at Saint Peter's University, I have tried to incorporate anxiety-reducing, esteem-building methods but have yet to focus specifically on self-efficacy building. At the same time, I feel that I have become complacent about my success in achieving that goal. This past year, therefore, my research has sought out reports from projects that tested specific approaches to improving self-efficacy in adult mathematics classrooms.

Self-Efficacy

In the United States, one of the traditional books read to children is the tale of *The Little Engine That Could* (Piper, 1930). The plot centres around a small, middling engine who is asked to transport a train of Christmas toys over the mountain when the stronger or flashier engines have turned down the task as being below their lofty status. While the little engine expresses trepidation at the prospect, his positive attitude and willingness to work hard results in success. The little engine triumphs because of his strong sense of self-efficacy.

It was Albert Bandura who first formalized the definition of self-efficacy and spent the bulk of his career exploring the impact of its presence, or absence, on the completion of a task. According to Bandura (1997), 'Perceived self-efficacy refers to beliefs in one's capabilities to organize and execute the courses of action required to produce given attainments' (p. 4). He recognized that 'efficacy beliefs regulate aspirations, choice of behavioral courses, mobilization and maintenance of effort, and

affective reactions' (p. 5). As stated earlier, many studies that investigated predictors of success in adult mathematics classrooms found that the strongest predictor was student self-efficacy. The belief that working hard and persisting through challenging assignments is within their power, leads students to succeed in the mathematics classroom.

While following the thread of Bandura's work through the research reports, the extended theory of Carol Dweck figured prominently. Dweck has researched and written extensively on what she calls 'mindset.' Although most of her publications are scholarly works, she has written for general audiences and addressed the impact of parents, teachers, and coaches on those in their care. Dweck asserts that 'the view you adopt for yourself profoundly affects the way you lead your life. A fixed mindset believes that your qualities are static. A growth mindset believes that your basic qualities are things that you can cultivate through your efforts' (Dweck, 2006, pp. 6–7). The common belief that the mathematical ability of an individual is set at birth is a clear example of fixed mindset at work in our society. By sharp contrast, in cultures where it is believed that everyone has the ability to do mathematical tasks and that ability is burnished by persistence and hard work, success is expected.

Dweck suggests strategies that promote movement from a fixed mindset to a growth mindset. These include:

- Establish a growth environment—the student is a developing person.

- Focus on processes—strategies, effort, choices.

- Offer constructive criticism that helps the student understand how to fix something.

- Set high standards and help the student reach them. Present topics in a growth framework and give students process feedback.

- For slower students, try to figure out what they do not understand and what learning strategies they do not have.

- Apply the growth mindset to your own teaching. (Dweck, 2006, pp. 205–206)

Research Linking Self-Efficacy and Success

Dissertation Research

Five doctoral dissertations found a link between self-efficacy and course success. Yoshida found that mathematics performance was significantly correlated with self-efficacy scores in a study conducted with adult university students (Yoshida, 2000). Schneider looked at the relationships between anxiety, self-efficacy, and performance in a graduate statistics course. He found an inverse relationship between statistics anxiety and statistics self-efficacy. There was no significant relationship between self-efficacy and performance, although there was a weak positive correlation between anticipatory self-efficacy and performance (Schneider, 2011).

In a study of university developmental mathematics students, Grassl found that regardless of other variables or demographic characteristics, self-efficacy continued to be a significant factor with significant predictive utility. Self-efficacy, elaboration, intrinsic motivation, extrinsic motivation, and mathematics anxiety all had a significant effect on students' success in the developmental mathematics course (Grassl, 2010). Wright, in a qualitative study that examined the experiences of developmental students who became mathematics teachers, found that affective qualities, including self-efficacy, were greatly enhanced by a teacher or mentor. He states, 'for some developmental mathematics students, a sense of self-efficacy stems from a relationship and connection with the developmental mathematics teacher more so, even, than with the material being learned. There is a desire to be like the teacher who helped them and to have that kind of impact themselves' (Wright, 2008, p. 134). Watts completed a study that found that age, mathematics anxiety, and mathematics self-efficacy were related, but mathematics self-efficacy was the only predictor of math performance. This was the only self-efficacy dissertation that looked at adult basic education students (Watts, 2011).

Journal Articles

Two journal articles specifically addressed an adult population. Hollis-Sawyer, in a study of nontraditional undergraduates, found that older (40+) men and women had higher levels of math anxiety and lower levels of self-efficacy than a younger cohort (18–39), yet there were no significant performance differences on a test of elementary algebra and geometry (Hollis-Sawyer, 2011). Goodwin examined gender differences in mathematics self-efficacy and testwiseness for adult learners engaged in undergraduate mathematics courses. 'Having self-confidence in general and high self-efficacy in particular, could make a substantial difference for the adult learner in undergraduate mathematics. Better understanding the relationship of gender with this idea of mathematics self-efficacy would help teachers to be more effective in their classroom management as well as assessment' (Goodwin et al., 2009, p. 37). It is noteworthy that there was not a statistically significant gender difference found in mathematics self-efficacy (Goodwin et al., 2009).

Four studies were set at the university level, a legally adult population in the United States. Jacobs, Prentice-Dunn, and Rogers conducted a study with university undergraduates solving difficult or insoluble anagram puzzles and found that self-efficacy expectancy had a strong effect on persistence. They concluded: 'The potency of efficacy expectancies in the present study clearly indicates the importance of emphasizing and enhancing perceptions of personal efficacy in situations requiring persistence. This may provide the cognitive stamina needed to persist in the face of difficulty' (Jacobs et al., 1984, p. 345).

Bates, Latham, and Kim, examined pre-service teachers' mathematics self-efficacy and mathematics teaching efficacy and compared them with their mathematical performance. They concluded that 'pre-service teachers who feel confident about their ability to solve mathematics tasks and do well in mathematics courses are more likely to feel confident in their ability to teach mathematics to children' (Bates et al., 2011, p. 328). And went on to say, 'Teacher preparation programs must examine their general education mathematics expectations along with their mathematics pedagogy courses to identify opportunities to modify curricular expectations that allow pre-service teachers' hands-on experiences to build their efficacy in regard to teaching mathematics' (p. 332).

In a study in Turkey, Erdoöan (2010) examined pre-service teachers' self-efficacy beliefs about the use of computers in mathematics education in their future classrooms. Erdoöan found that there are significant relationships among self-efficacy, outcome expectations, interests, and intentions, as was predicted by previous studies. Renninger, Cai, Lewis, Adams, and Ernst (2011) studied teacher professional development in an unmoderated online workshop. Their findings indicate that while three clusters of motivational profiles could be identified (low interest, low self-efficacy, and less math; low interest, high self-efficacy, and more math; high interest, high self-efficacy, and more math), whether the teachers continue to participate appears to be related to the structure and content of the workshop, not just these profiles. The authors concluded that the potential of hypermedia lies in its designers' abilities to support participant stake by providing for multiple representations or ways into thinking and working with the disciplinary content of the given online learning—design that both accommodates and supports those with differing strengths and needs (Renninger et al, 2011, p. 246).

The seeds of a sense of self-efficacy appears to be sown in elementary school and either flourish or wither in the middle school grades (students ages 11 to 14 in the United States). Friedel, Cortina, Turner, and Midgley studied the changes in self-efficacy beliefs and mathematics as students move from elementary school (age < 12) to middle school. They studied the goals math teachers emphasize on changes in students' self-efficacy beliefs across the transition from sixth to seventh grade, taking into account students' perceptions of the goals parents emphasized for them in mathematics as well as the personal goals children adopted before and after the transition. They found that teachers can substantially influence the efficacy beliefs of their students simply by placing emphasis on learning and improving understanding in mathematics, in effect ameliorating low self-efficacy beliefs that may have resulted from experiences in previous math classes (Friedel et al., 2010). Tella (2011), in a study conducted with secondary school students in Nigeria, examined the relationship of self-efficacy to gender, age, and mathematics achievement. He found significant differences in self-efficacy between male and female students as the students aged from 11 to 18 and between self-efficacy and achievement.

In a study with secondary school students in the Netherlands, Vrugt, Oort, and Waardenburg tested a causal model demonstrating how mastery and performance goals affect motivational variables (self-efficacy and downward comparison) and how these variables influence academic achievement through affect. Mastery goals affected self-efficacy, and self-efficacy predicted achievement through affect. Performance goals affected downward comparison, which predicted achievement through affect. In line with the multiple-goal perspective, the results showed that individuals do simultaneously pursue mastery and performance goals (Vrugt et al., 2009).

In a study conducted in Iran, Nasiriyan, Azar, Noruzy, and Dalvand explored the impact of self-efficacy and task value on students' achievement goals. Self-efficacy had a direct effect on mathematics achievement. That is, a belief about math ability is an important factor in math achievement. The study results also indicated that mastery goals influenced mathematics achievement indirectly through the mediation of effort. Mastery goals' influence was positive on effort (Nasiriyan et al., 2011).

Williams and Williams explored the reciprocal determinism of self-efficacy and performance across 33 countries using the Program for International Student Assessment (PISA) 2003 Mathematics Achievement Test. The authors suggest that 'the reciprocal determinism of mathematics self-efficacy and performance may well be a fundamental psychological process that transcends national and cultural boundaries. It does not appear to work in exactly the same way in all nations and did not hold true at all in four of the nations.… The substantial and consistent negative effect of gender on self-efficacy is of particular interest. Other things being equal, girls reported lower levels of self-efficacy than did boys, consistent with the observed differences among country means' (Williams and Williams, 2010, p. 463).

Intervention Studies

Dissertation Research

Far fewer studies investigated intervention strategies that foster self-efficacy in the adult mathematics classroom. Three dissertations described approaches that did so. Rowland, in a study of 15 adult undergraduates, found that the following teacher behaviors promoted self-efficacy:

- *Verbal persuasion*, in which the instructor gives a clear statement of his/her philosophy and expectations, continually offers positive reinforcement, and encourages questions at all times.

- *Emotional arousal* is mitigated by a relaxed classroom environment, a patient teacher, content relevant to student lives, and the use of manipulatives.

- *Vicarious learning* was supported by the manner in which course material was presented and by both the teacher and peers modeling successful critical-thinking and problem-solving strategies (Rowland, 2004).

Zielke, in an intervention with three developmental students, applied a sports-coaching methodology to heighten their self-efficacy beliefs, using the letters of the word 'champ' to delineate the various steps:

- **C** is for **c**ue words that remind athletes to do certain things at certain times in a game/match.
- **H** comes from focus on the **h**ere and now when setting achievable goals.
- **A** stands for **a**rousal control—the need to control emotions during competition.
- **M** comes from **m**odeling and **m**ental imagery.
- **P** has three roots—**p**raise, verbal **p**ersuasion, and **p**ositive self-talk (Zielke, 2001, pp. 4–5).

Tintera (2004) explored the use of graphing calculators to promote self-efficacy in an algebra class for adult students. Her results showed that the use of a graphing calculator successfully and positively promotes individuals' self-efficacy in mathematics; this was particularly true for students who favored a visual learning style.

Journal Articles

Two journal articles recommended approaches that further the growth of self-efficacy. Parks adapted a phrase from language arts and spoke of *genres* of teaching. She offered a menu of teaching strategies for use in the elementary classroom that apply equally to mathematics education at the adult level:

Genres of Teaching in a Classroom

- Mathematical discussions
- Game shows
- Group work
- Individual student math book work
- Cross-examination
- Presenting work publicly
- Individual student–teacher conversations
- Games
- Journal writing
- Tests/quizzes (Parks, 2009, p. 606)

Baker and Campbell videotaped undergraduates in an advanced mathematics class tasked with the completion of three proofs. The authors focused on group aspects and found that success was predicated on a combination of ability, self-efficacy, and collective efficacy. Based on their observations, they recommended that the instructor should assign groups and include high ability and high self-efficacy in every group. S/he should identify the steps in the problem-solving process, model the process, and give students plenty of opportunities to practice using it. The instructors need to provide feedback about individual and group performance to keep students from spending a lot of time and effort learning erroneous information or misapplying concepts. It may be necessary to intervene by regrouping or tutoring group members about group dynamics and providing feedback about their group skills (Baker and Campbell, 2005).

Finally, I return to Carol Dweck and her concept of mindset. In order to assist students who may enter the classroom with a fixed mindset to transition to a growth mindset, she suggests that each of us establish a classroom environment that promotes student development and reminds us that we too are works in progress and need to apply the growth mindset to our teaching (Dweck, 2006).

Plan for Academic Year 2014–2015

A chief focus for my instruction in the coming academic year will be the fostering of self-efficacy. I teach three different student audiences during the academic year: pre-service teacher candidates, business and social sciences majors, and undergraduate and graduate students seeking a middle school mathematics certification. In each class this year I plan to incorporate the suggestions from Rowland and devote the first class to sharing my philosophy of teaching and clarifying expectations for both the students and myself. The second class will focus on reading a mathematics textbook effectively. Different strategies will be employed in specific courses, as described below.

Math for Educators

Students in this course often, though not exclusively, have weak mathematical backgrounds. They need to build their self-efficacy as they prepare to teach in the elementary schools. Parks' concept of genres is appropriate for this class. It is important that pre-service students experience the full range of teaching/learning methods they will be expected to utilize when 'real' and that I model their use. Rowland's use of journals would be suitable for this group. I may replace the weekly quiz with a prompted, reflective journal entry.

Calculus for Business and the Social Sciences

This class has a disparate group of students—some quite strong and others who struggle to keep up. The advice about group composition offered by Baker and Campbell suggests that the administration

of a self-efficacy instrument early in the semester would help to compose effective groups. All of Rowland's suggestions are appropriate for this group, although tactile manipulatives may not be useful. Certainly connections to real-world situations reinforce the utility of calculus in the various disciplines of the students.

Functions for Middle School Teachers

This course is similar to Mathematics for Educators, although the students are often classroom teachers. They are truly 'adult' and often bring an awareness of the importance of mathematics in everyday life that enhances classroom discussion and group work. Rowland's problem solution analyses and journals would be appropriate for this course. The research linking mathematics self-efficacy and mathematics teaching self-efficacy makes clear the importance of effective efficacy-building experiences for these pre- and in-service students.

Conclusion

There is a substantial evidence base supporting the importance of self-efficacy for success in the adult mathematics classroom. Unfortunately, there is a paucity of studies that report the results of interventions striving to enhance self-efficacy as a means to promote student success. With this paper I hope to open a dialogue that will encourage instructors to further the success of their students by utilizing classroom methods that support the growth of self-efficacy.

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Teaching Maths for Medical Needs: A Working Partnership between Teachers and Healthcare Professionals to Support Children and Young People with Type 1 Diabetes with Concepts of Ratio

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Abstract

Sheffield Hallam University and Sheffield Children's Hospital run a week-long 'Teaching Skills for Healthcare Professionals' programme in England. This programme supports healthcare professionals in working with children and young people on managing their medical conditions, particularly type 1 diabetes mellitus. The programme includes a session on ways to teach the concept of ratio. This paper explores the interface between the ways in which we use mathematics in our work or everyday lives and the ways in which we might explore the same mathematics as teachers in a classroom and/or in a healthcare environment. The approaches used on the programme are critiqued with reference to existing literature on mathematics and healthcare, subject pedagogical knowledge and visualisation and mathematics.

Keywords: mathematics, healthcare, programme approaches

Introduction

The theme of this year's ALM conference in Bern was 'Maths inside and outside the classroom'. This paper develops that theme in the context of healthcare.

At Sheffield Hallam University in England, we run a five-day intensive programme for healthcare professionals (HCPs), mainly nurses, dieticians and doctors. The aim of the course is to support the participants in delivering health education to children and young people with complex health needs. The course originated from national recommendations by the National Institute for Care and Excellence (NICE) for the need of paediatric diabetes teams to be appropriately trained to develop and deliver structured diabetes education to children and young people (NICE 2004). This entails teaching them how to match their insulin dose to their carbohydrate (food) intake. Improving diabetes education reduces the risk of diabetes related complications, such as retinopathy, neuropathy and nephropathy, in later life. For the purpose of this paper the needs of the health care professional in relation to caring for children and young people with type 1 diabetes mellitus will be discussed in relation to maths. However the course has shown a need in other health care specialities, such as Cystic Fibrosis, Haematology, cancer, late effects and general children's nurses from the ward and as such they are also beginning to join the Sheffield Hallam University programme in increasing numbers.

Teaching Skills for Healthcare Professionals - the programme

The five-day programme includes the following:

- A focus on generic teaching skills, e.g. planning active learning, assessing learning, motivating learners, exploring teaching styles – related to working within a healthcare context.
- A one-day placement in a school setting to experience how young people aged 11-16 years old learn and are taught in schools.
- Experience teaching in a secondary school whilst being peer reviewed.
- Reflecting on and sharing experiences with other course participants and tutors.
- Developing age appropriate curricula and teaching aids.
- A morning of mathematics, with a focus on how to teach dose adjustment of daily medication. This is to support children and young people with type 1 diabetes mellitus in thinking about how much carbohydrate is in any particular portion of food, and how much insulin this needs in order to maintain optimal blood glucose levels.

The programme was developed by Julie Knowles, a research nurse (and co-author of this paper), and Anne Michael, a senior lecturer at Sheffield Hallam University. It came about as a collaboration between Sheffield Hallam University and Sheffield Children's Hospital. Previously, Julie had been involved in developing an educational programme for young people aged 11-16 years with type 1 diabetes mellitus, entitled Kids In Control OF Food (KICk-OFF) Knowles et al (2006) and Price et al (2013).

KICk-OFF is a 5 day structured intensive education programme teaching children and young people the skills to adjust their insulin dose to their carbohydrate intake as well as managing their treatment during times of illness, exercise and social activities. This involves using complex mathematics skills at least four times a day.

A visual representation for our model of delivery for the five day healthcare practitioner programme at Sheffield Hallam University can be seen in Figure 1 below.
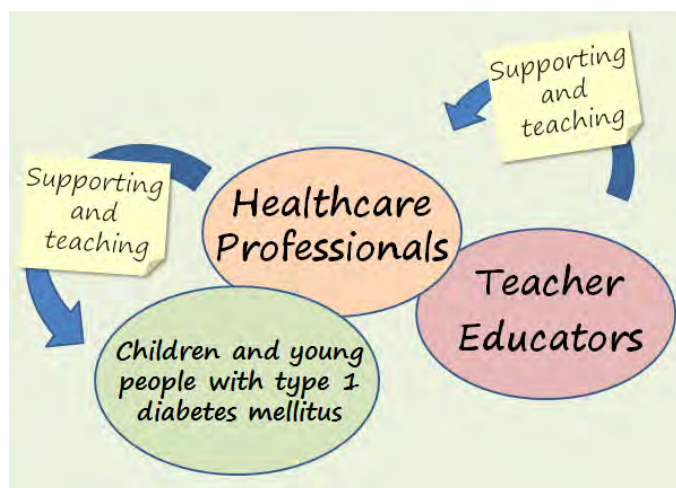
Figure 1. Delivery model for 5-day HCP programme

The team at Sheffield Hallam University supports and teaches healthcare professionals within the university campus ('inside' the classroom), working to develop their skills in teaching and supporting learning. The healthcare professionals, within their own roles, support and teach children and young people with type 1 diabetes mellitus and their parents/carers how to manage their medication and diet in order to regulate their blood sugar levels. This instruction and support takes place both in formal settings (usually within a clinic) and at the homes of the learners ('inside' and 'outside' the classroom).

The children and young people with type 1 diabetes mellitus and their parents/carers in turn have their own practices in regulating their medication and diet in their own lives ('outside' the classroom). This combination of education both inside and outside classroom or formal settings presents its own opportunities and challenges. Focussing on the mathematical part of the programme, this paper sets out to explore some of these.

The mathematics session currently takes up one morning of the course, although references are made to calculations, measurements and other related areas throughout the entire programme. The content of the taught 'maths' session is as follows:

Exploring Weights and Carbohydrate Content of Real Food

This involves each participant selecting a food item from a selection of packets of real food (e.g. rice, pasta, tins, and sachets of sauce mixes). We then ask the participants to estimate the weight of their food item. They then check their estimates and line up in order of food weight, holding their chosen item in front of them for all to see. We then ask them to re-order themselves according to how much carbohydrate per 100g they think is in the food. This involves some discussion. Once they have done this they check their carbohydrate estimate on the nutrition label and re-order themselves in the correct order. We then discuss any surprises, along with how confusing it can be to conceptualise carbohydrate content, especially when it is given as a ratio (per 100g).

Different Ways of Calculating Ratios

Here we look at carbohydrate to weight ratios and insulin to carbohydrate ratios, and share and discuss mathematical methods for working these out.

Working With Visual and Tactile Representations of Ratios

This part of the session involves playing with coloured modelling dough and cut out shapes in order to create visual and tactile ways of presenting carbohydrate to weight and carbohydrate to insulin ratios.

Compiling a Chart

We ask participants to create a chart showing the carbohydrate amounts and related insulin doses needed for a particular person, for different types of food. They then carry out a short, informal role play, with one person acting as the healthcare practitioner (HCP) and the other as the person with type 1 diabetes mellitus. The HCP asks their partner questions about the charts in order to assess their understanding of the processes and concepts involved. During this, the session trainer circulates and makes notes of any good explanations or questions, or any areas where confusion may arise, and these are discussed as a whole group. Figure 2 below shows examples of resources used in this activity.

Figure 2. Examples of resources used in 'chart' activity

Making Cheesecake

- The 'maths' morning ends with a practical session involving making cheesecake and working out the carbohydrate needed per portion. This is followed by a whole group plenary to share experiences and discuss issues that may arise when carrying out a similar session with learners.

All of the above content was developed by the course team (Martyn Edwards, Anne Michael, Julie Knowles and Rachel Stone), in response to the identified professional development needs of the healthcare professionals attending the course, in relation to educating children and young people with type 1 diabetes mellitus.

Exploration of Some Sample Activities

We will now explore some examples of activities used in the session, followed by a look at some of the general theory and research underpinning the rationale behind them.

Sample Activity 1

This looks at how to work out how much carbohydrate in a portion of food. Suppose you had a bowl of cornflakes for breakfast (a popular breakfast choice in the UK). A look at the label on the cereal box or a quick internet search will show that cornflakes typically have 85g carbohydrate for every 100g total weight. Suppose you weigh your portion of cornflakes and it comes to 40g. How much carbohydrates will there be in the 40g of cornflakes in your cereal bowl? On the Teaching Skills for HCPs programme, we use a similar calculation derived from one of the food packages selected by the participants at the start of the session.

This is what is known as an 'isomorphism of measures' problem. According to Vergnaud (1983) there are five ways to solve this. In our HCP session we focus on three of these, as illustrated in Figure 3 below.

Different ways to solve proportional reasoning problems

Scaling

Carbohydrate	Total weight
85g	100g
?	40g

Unitary

85 g carbs per 100 g

0.85 g carbs per 1 g

0.85 × 40 g per 40 g

Three point formula ? = (85 × 40) / 100
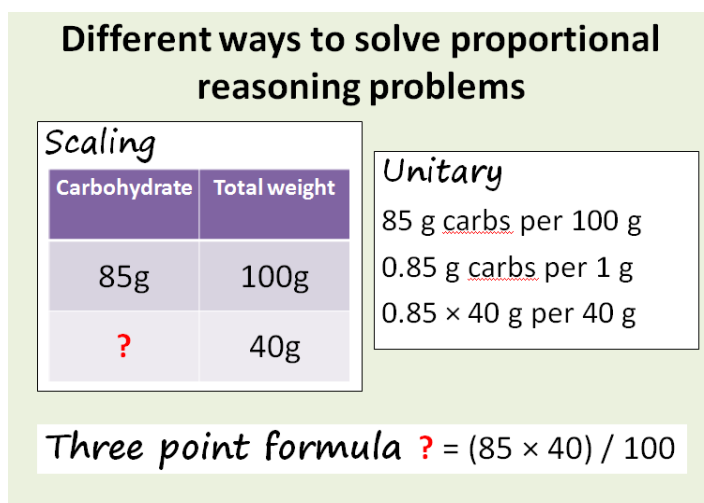
Figure 3. Different ways to solve proportional reasoning problems

The 'scaling' method involves using a visual representation of the quantities given, and scaling the amounts up and down in the required ratio until the desired quantity is reached. This method keeps the numbers in the calculation close to their original meanings and is therefore fairly intuitive in nature (Wright 2009).

The unitary method involves finding out how much carbohydrate there is in 1g of cornflakes, and then multiplying this up to the desired quantity. Finally, the 'three point formula' (also known as the nursing formula) uses what is essentially an algebraic method, manipulating the given expression in order to isolate the unknown.

It is worth noting that in the HCP session we do not 'present' these methods as such, rather we ask the participants to share their different ways of working out the amount of carbohydrate in a 40g portion of cornflakes (or whatever the given question is). Each of these three methods is usually mentioned at some point.

Which is the 'right' or the 'best' method to use? Which is the best method to teach? There has been much research and debate around the use of calculation methods to work out medicine doses (e.g. Coben and Weeks 2013, Wright 2008, Hoyles et al 2001). Although our example here is about carbohydrate amount, the same techniques apply when calculating insulin doses for given amounts of carbohydrates, working to a set insulin-carbohydrate ratio (ICR). Traditionally, the 'nursing' formula has been used in nurse education programmes, and indeed, Hoyles et al (2001) found that the nurses in their study relied heavily on this formula in formal settings such as written tests. However, some research indicates that other methods, such as the scalar method, or the use of an artefact such as a syringe, can take precedence in a practical clinical setting (Coben and Weeks 2013, Wright 2008, Hoyles et al 2001).

The participants in the last cohorts on the Teaching Skills for HCPs programme were mixed in their views on this. From approximately 30 participants, some said that they relied on the formula but liked the scalar method as it made more sense to them. Others said that they preferred the unitary method as it worked every time regardless of how difficult the numbers involved were, providing you had a calculator. In the cheesecake making session, the unitary method appeared to be the most popular. Further anecdotal evidence suggests a range of approaches according to the individual concerned, including other examples where scalar methods or syringe-based methods are preferred.
Since this information was not gathered in a formal research context, it is not possible to generalise from it. However, what is apparent is that different people have different preferences when it comes to calculation methods for working out drug dosages or amounts of carbohydrate in food portions. What are the implications of this from a pedagogical perspective?

On the one hand, we feel further research is necessary in the area of what social practices children and young people with type 1 diabetes mellitus and their parents/carers are engaging with when calculating carbohydrate amounts and insulin doses, including what artefacts they are using to assist them with this (e.g. insulin pumps, calculators, weighing scales, mobile phone apps). This will have an impact on the methods that they are being taught by healthcare professionals. On the other hand, it is important for healthcare professionals to be open to working with whatever method of calculation the learner is most comfortable with. This involves a considerable amount of what Shulman (1986) terms 'content pedagogical knowledge'. For example in this context this might include:

- A range of different methods of calculation using ratio and proportional reasoning, including formal and informal methods, those used at home or in everyday life (social practices)
- An understanding of the mathematical concepts involved
- The degree of accuracy that is acceptable/necessary
- The use of a range of artefacts to help with this
- How to assess, in a non-threatening way, the existing knowledge or the method preferred by the learner. Several of our course participants said that it was difficult to do this because many learners, especially parents, were unwilling to expose their lack of knowledge
- How to judge where or when a different method of calculation might be more appropriate
- How best to introduce a new method in a way that the learner is most likely to understand, e.g. using visual or tactile representation, what language to use, what artefacts to use

This is a daunting amount of pedagogical content knowledge. However, it is likely that much of this knowledge is already being employed by professionals in healthcare education contexts. Further research is therefore needed to explore this area further. However, the principle that it is inadvisable to go in with a 'one approach fixes all' method is borne out not only by the research literature but also from the feedback from the HCPs on our courses.

Sample Activity 2

In discussion with the healthcare professionals, the following points arose:

- Some were not interested in the conceptualisation of ratios but simply wanted a formula that worked, while others were attracted to methods that were more visual or intuitive and less formulaic.
- Several wanted to know how to explain the concepts of ratio and proportional reasoning to those learners who struggled, including children who might not yet have met these concepts at school, or parents/adults who had difficulties with mathematics.

As mentioned, Coben and Weeks (2013) document the use of what they refer to as '*context-bound artefacts'* or '*functional objects'* in calculating drug doses. They mention the use of syringe in particular (and in fact explore the use of a virtual syringe on a computer). On the Teaching Skills for HCPs programme, we use the following context-bound artefacts: food items, weighing scales, pictures, calculators, all of which helped participants to conceptualise and solve proportional reasoning problems. Such objects may help to bridge the gap between conceptual understanding and the use of pre-set algorithms.

A related term to 'artefact' in this context is the notion of a 'boundary object' proposed by Star and Griesemer (1989). These objects may be abstract or concrete. They have different meanings in different social worlds but their structure is common enough to more than one world to make them recognizable, a means of translation (p. 393).

In finding a way to represent the concept of carbohydrate to total weight ratio, or carbohydrate to insulin ratio, we were looking for tactile and visual resources that could act as 'boundary objects' that could carry conceptual meaning for individuals from a classroom-based mathematical context to an everyday, 'real-world' context. To this end we modelled the use of diagrams and 'manipuables', cut out tactile objects that could represent grams of carbohydrate and units of insulin. An example is shown in Figure 4 below, with circles and parts of circles to illustrate units of insulin, and strips of squares to illustrate grams of carbohydrate.

Figure 4. Resources to illustrate units of insulin and grams of carbohydrate

Once cut out (and even laminated), these could then be placed in charts and tables to show how many units of insulin match given amounts of carbohydrate, and vice versa. They could then be glued down as a permanent record, or placed on a laminated chart with a 'velcro' strip attached, to allow them to be moved around (manipulated) for different insulin-carbohydrate ratios. Note that we used tangible materials rather than virtual simulations. This was because (a) we wanted to reflect the availability of technology-related resources within the environments in which our HCPs were working, and (b) we felt that the tactile nature of the resources themselves added something to the learning experience (Prinn, 2013, p. 174).

These resources were well-received by our HCP course participants. However, more investigations are needed to find out how they are received by the people with type 1 diabetes mellitus, who are themselves being trained and supported by these HCPs. At their best they could act effectively as boundary objects and visualisations, at worst, their very context-free nature may detract from their use, as might the fact that they are not learner-generated but 'teacher-generated'. Indeed, research on the use of visual aids and diagrams in solving mathematical problems indicates that their effectiveness is not universal (e.g. Booth & Thomas, 2000). Nevertheless, the responses from the participants on the Teaching Skills for HCPs programme were encouraging. Further fieldwork may uncover other types of boundary object or artefacts used by children, adults and young people in conceptualising and calculating with insulin to carbohydrate ratios.

Sample Activity 3

As noted at the beginning of this paper, our work at Sheffield Hallam University with the HCPs took place within the university itself, and some (though not all) of the HCPs' work with people with type 1 diabetes mellitus takes place in a relatively formal educational, clinical setting. In presenting the theory of situated learning, Lave (1993) asserts that

> We have come to the conclusion, as McDermott (1993) suggests, that there is no such thing as 'learning' *sui generis*, but only changing participation in the culturally designed settings of everyday life. Or, to put it the other way round, participation in everyday life may be thought of as a process of changing understanding in practice that is learning. (Lave in Illeris, 2009, p. 201).

In other words, learning is inextricably linked to the context in which the learning takes place. It is true that home visits between HCPs and their patients do take place, and thus provide an ideal opportunity for learning within an authentic context. However, this is not always possible due to time and budgetary constraints, and so teaching happens in other settings too.

The challenge, then, is how to address what Coben and Fitzsimons (2009) refer to as 'problems in the transfer of learning between the classroom and the workplace', or, in this case, between the classroom or clinical setting and the home or everyday environment. Gulikers et al (2004) in Coben and Weeks (2013) present a framework for assessment that includes the use of authentic tasks, using physical and social contexts as close to the 'real thing' as possible. To this end, we include a practical session on the Teaching Skills for HCPs programme, which involves the making of cheesecake. This meets the framework above by providing an activity with a meaningful purpose (making and eating cheesecake and calculating the carbohydrate amount per portion), in real time, using authentic ingredients and tools, and involving collaboration and social interaction with peers. Research on the KICk-OFF programme (Knowles et al, 2006) shows that a practical cookery session such as this can be motivating and stimulating for young people. In fact, during the development of the KICk-OFF programme, focus group analysis revealed that one of the most important themes of an education programme for children and young people was to include meaningful, practical sessions Waller et al (2004).

Similarly, on the HCP course, the activity was well received, and much of the learning took place in the informal discussions that occurred during the session, for example, conversions between imperial and metric units, techniques for measuring and estimating amounts, degrees of accuracy necessary and so on. While this is not an activity that everyone would necessarily engage in at home or in their own lives, it served as a model, both for the HCPs as a teaching technique and for learners as a recipe idea, and thus would hopefully be replicated in the participants' own practices. Once again, further research would be useful in ascertaining the effects of such a session in practice in the community.

Summary and Conclusions

Our experiences of planning and facilitating the mathematics sessions on the Teaching Skills for HCPs programme, and our analysis of the examples discussed have led us to propose the following hypotheses:

- To teach and support children and young people with type 1 diabetes mellitus and their parents/carers to calculate carbohydrate intake and insulin dosage, it would appear that some 'content pedagogical knowledge' is necessary (Shulman, 1987).
- The use of boundary objects or artefacts is of key importance in the transfer of learning from a clinical or classroom setting to a home based or everyday setting. Further research needs to be done on the use of and social practices around such objects by people with type 1 diabetes mellitus.
- The use of authentic assessment for learning tasks and a holistic approach are essential in developing learner understanding of proportional reasoning problems in this context. This suggests that the 'condensing' of mathematics into one morning of a 5 day course may need to be reviewed, and that the mathematics may need to be more embedded into the programme as a whole. Similarly, the value of a 5 day intensive programme is also something that could be researched further, and other models of delivery investigated (e.g. distance learning, peer support groups, 'training the trainers' for cascaded delivery, practitioner research).

  Another, more general conclusion, based on our experiences of the programme, our respective professional expertise, our own research and our review of the related literature include the following:

- Diabetes-related education and training should not be limited only to those with diabetes. A whole-society approach is needed, in particular schools and other educational settings. For example the KICk-OFF team recognised that children and young people knew what they had to do to maintain optimal health. However they did not have the skills to succeed, especially when away from home. The KICk-OFF course introduced practical ways for them to work out how best to manage their diabetes in changing environments. However it is also recognised that children and young people need the support from their peers and the school environment to be successful in their care (Wagner et al, 2006). Similarly, more public awareness is needed to create a more conducive environment for learning and applying knowledge about diet and medication in relation to diabetes.

To this end, we would like to see our 'Teaching Skills for HCPs' model change from the one illustrated in Figure 5 below:
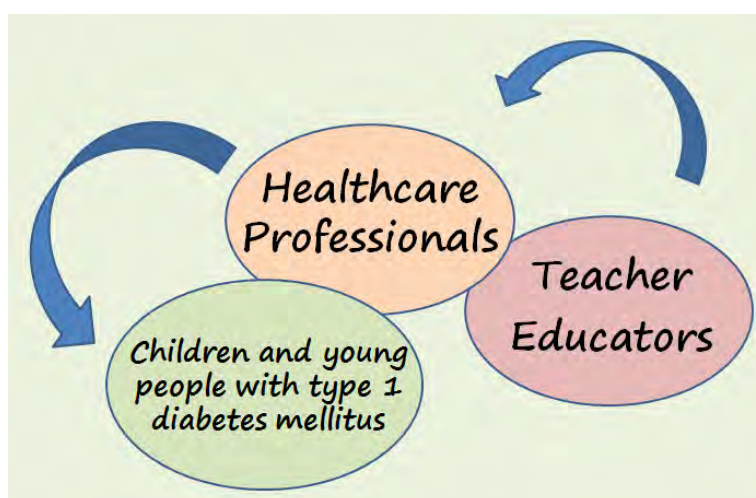
Figure 5. Our current model of delivery

To something more the one shown in Figure 6 below:
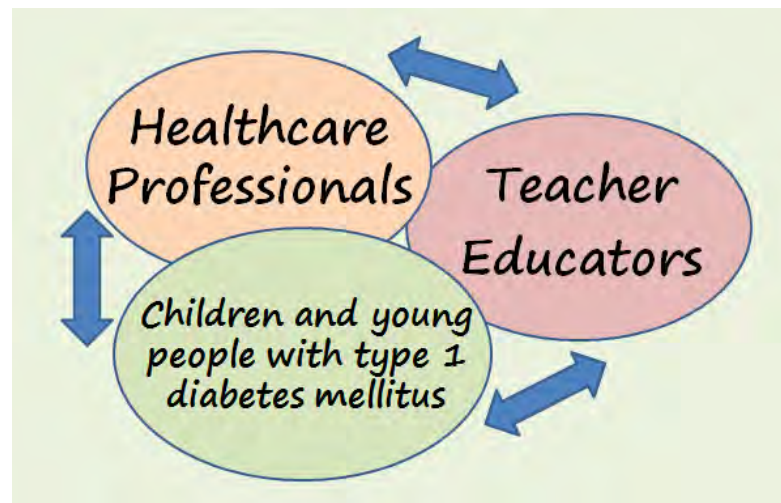
Figure 6. Our proposed model of delivery

If we are to support HCPs from diabetes and other specialities in their educational work more effectively, further research is needed into not only their professional practices but the social practices of the people they are supporting, in relation to their medical conditions and their management.

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Section 2.b.

Presenter Power Point Presentations, Abstracts and Posters

The Non-mathematician and the Spreadsheet: The Case of the Missing x

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Tools for Teaching "Everyday Mathematics"

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Workshop „Everyday mathematics"

Didactics:
Didaktisch-methodische Überlegungen:

➢ Participant oriented, i.e., consider participants' resources and needs, flexible lesson planning, respect different ways of finding solutions

➢ Teilnehmerorientiert, d.h., Ressourcen wahrnehmen, Fehler tolerieren, Bedürfnisse wahrnehmen, flexible Gestaltung des Unterrichts, Lösungsstrategien der Teilnehmer/innen respektieren

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Workshop „Everyday mathematics"

Didactics:
Didaktisch-methodische Überlegungen:

➢ Build motivation: enjoy dealing with numbers, play and practise, learning by doing, relating it to the person and his/her way of life, from the „close" to the „far" as intercultural challenge

➢ Motivation aufbauen: Lust am Umgang mit Zahlen entstehen lassen, praktisches und spielerisches Üben, Handlungs-orientierung, Bezug zur eigenen Person und Lebenswelt, vom „Nahen zum Fernen" als interkulturelle Herausforderung

Slide 11

Workshop „Everyday mathematics"

12 chapters of teaching material:

12 Kapitel Lehrmaterial:

0 Numbers	0 Zahlen ZLSS
1 Climate and temperature	1 Klima und Temperatur
2 Getting around in Basel	2 Orientierung in Basel
3 Money	3 Geld
4 Weight	4 Gewicht
5 Measures of length	5 Längenmasse
7 Measures of capacity	7 Hohlmasse
8 Shopping	8 Einkaufen
9 Job administration	9 Jobadministration
10 Budget	10 Budget
11 Speaking–Thinking–Logic	11 Sprechen–Denken–Logik
12 Operations	12 Operationen

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Supporting Numeracy Teaching and Learning in Adult as a Second Language (ESL)

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Some Issues - Implication for STEM

STEM and URMs

- ❏ Many adults in the adult education system have children in the K-12 system who are failing mathematics
- ❏ Many adults do not have sufficient capacity in math to help their children
- ❏ Many children do not have sufficient capacity in math to excel in science, technology, engineering, and mathematics (STEM)

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Some Issues - Implication for STEM

STEM and URMs

- ❏ Many minority children typically do not develop a math identity (Berry & McClain, 2009; Martin, 2007)
- ❏ Building a strong math identity (Martin, 2007) is particularly important among women, African Americans, and Latinos given their lack of representation in STEM educational programs.
- ❏ Minorities do not "see" themselves in STEM - Lack of Science Identity (Carlone, Cook, Wong, Sandoval, Calabrese Barton, Tan & Brickhouse, 2008)

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Some Issues - Implication for STEM

STEM and URMs

- ❏ The overall production of STEM doctoral degrees has fallen in the United States (Nettles & Millet, 2006).
- ❏ The decline in STEM degrees is extremely pronounced among URMs in graduate education
- ❏ URMs who fall outside of the predominantly White, masculine sociocultural norms of many STEM disciplines often experience feelings of marginalization and social isolation (Fries-Britt, Johnson, & Burt, 2013; Herrera, Hurtado, Garcia, & Gasiewski, 2012).

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Some Issues - Implication for STEM

STEM and URMs

❑ Many students, including URMs may have a different cultural understanding and use of math compared to the mathematics presented in most U.S. classrooms.

❑ Many URMs and ELLs are likely to have problems developing a science identity in terms of "fitting in" with the sociohistorical norms of most STEM disciplines

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Some Issues - Implication for STEM

STEM and URMs

❑ Few researchers have examined
  ❖ Numeracy among URMs in general
  ❖ Numeracy as a means of increasing students' math and science or identity
  ❖ Student motivation to enter STEM undergraduate and graduate programs in the United States

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Where are we?

❑ **2013 Research on Adult Learner Transition to Post Secondary Education** (small study)
* Support is provided to many adults, including ELLs to support their transition
* Many programs are modifying programmatic structure
* Challenges found in supporting ELLs

Findings from 2008 Adult Numeracy Initiative
❑ "Research needs to pay particular attention to instruction for adult ESL learners, who make up over 40 percent of students in adult literacy programs" (Condelli et al., 2006, p. 62).

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Where are we?

Program for International Assessment of Adult Competencies (PIAAC)

❑ Over 5,000 US adults between the ages of 16 and 65 were tested (by Organization for Economic Cooperation and Development (OCED).

❑ **Results**

❖ **Literacy** - United States statistically close to the PIACC international average

❖ **Mathematics** - The US was significantly lower than the PIAAC international average in math and problem-solving

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Reflecting on Past Recommendations

Statement by U.S. Department of Education

❑ "Since today's decisions are based on data, it is equally important for adult learners to develop and strengthen skills in mathematics, and for educators to identify what works best in developing those skills ......

❑ Mathematics skills are a gatekeeper for further education and training, and significantly affect employability and career options" (U.S. Department of Education, 2010, pr.1).
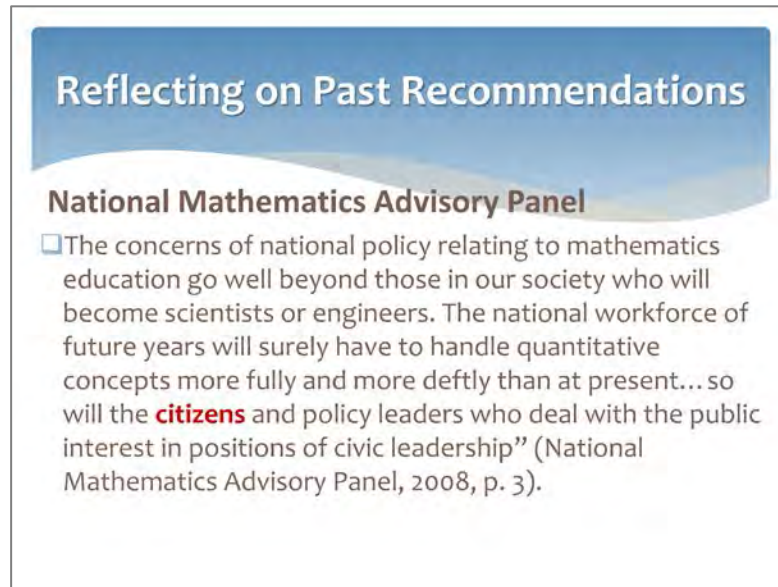
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Reflecting on Past Recommendations

National Mathematics Advisory Panel

❑ "Mathematics literacy is a serious problem in the United States. . . . National policy must ensure the healthy development of a domestic technical workforce with adequate scale and top-level skill. (National Mathematics Advisory Panel, 2008, p. 3).

On April 18, 2006, President Bush issued an Executive Order creating a National Mathematics Advisory Panel to advise him on the best use of scientifically based research for mathematics teaching and learning. See *The Final Report of the National Mathematics Advisory Panel* at http://www2.ed.gov/about/bdscomm/list/mathpanel/report/final-report.pdf.

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Learning Maths at Work through Trade Unions in the UK

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LEARNING MATHS AT WORK THROUGH TRADE UNIONS IN THE UK- RESEARCH BY BETH KELLY

WHAT MAKES IT DIFFERENT TO PREVIOUS LEARNING?

"Not what I would call a formal learning environment, very relaxed... small gathering... almost a social thing.... 38 year old man

"The information we are using is relevant to us. Last year we did asbestos deaths, it's relevant to us and our work." 54 year old woman

On Line Learning Mathematics

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Abstract

The primary presentation was based on our experience of teaching on-line introductory Probability course at the Community College of Philadelphia. The course structure and supporting computer systems were described. The discussion focused on challenges students would face in learning and understanding the course ideas and concepts while following the rigorous course time frame. Powerful on-line course tools such as interactive textbook, video materials, individualized help tools as well as the role of the instructor were discussed. The advantages of hybrid courses compared to strictly on-line ones were debated. The participants of the workshop shared their experience in using various on-line learning systems for independent and personalized learning and as a supplementary learning tool in the regular courses.

Toolkit Numeracy Concept

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Toolkit Numeracy Concept

Focus

The learning material is directed at course managers that need to support course participants in filling gaps in their knowledge of mathematical basics.

There are various reasons why insecurities arise in dealing with mathematical basics.

- The use of the knowledge lies far in the past and has to be re-activated
- The knowledge was not built up solidly. Instead overcoming tricks were developed, that don't allow a competent handling of the contents.

In both cases mere mechanical practise of the contents will not have a lasting effect. More importantly the corresponding basic concepts need clarifying, reactivating or improving. This improvement of understanding can for instance be supported with a discussion about the issue with suitable visualisations. On this basis the learning material "Toolkit" was developed.

The mathematical basics can be understood as tools that are helpful when overcoming certain everyday situations. When these tools are missing partly or completely, course managers are called for to improve their development.

The learning material at hand is to support the development of these "tools". The "tools" can be learnt separately. To that end individually applicable learning material to specific basics is available. The material can be used individually and has been designed for individualising lessons. The knowledge gaps of the course participants regarding mathematical basics can be very different. Whoever, for instance, has understood percentages profoundly, has no need to spend time with the derivation of calculating percentages. Persons unsure of themselves in this field can be supported with the learning material. Working with the learning material should lead to questions that help define the extent of the insecurities. In this way course managers can enter individually and very specifically into these topics. The learning material offers teaching suggestions.

Content of Learning Material

The learning materials are subdivided in three categories.

MS   **math-starter**

„math-starter" contains problems that lead to discussions and imply simple common calculations. These can be from newspapers, can contains current news,… or be initiated by the course participants.

D   **Determination of Knowledge Level**

The knowledge gaps or insecurities can be determined in two ways:

Determination of knowledge level in the form of problems:
The course participants solve problems from the chosen topics to determine the extent of the insecurities.

Determination of knowledge level in the form of a conducted discussion:
In the discussion the core of the mathematical basic concepts is crystallised to link the facts and to activate previous knowledge.

I **Improvement of Understanding**

In contrast to mechanically learned knowledge, knowledge based on understanding is more flexible and available long term. For this reason this part gains special significance.

With the learning material, opportunities are offered in which insecurities can be clarified in dialogue in between the course manager and course participants. Sketches show how to work with models, visualising props and structured problems.

EDU call notes for introduction in the classroom.
E.G. denote examples that can be incorporated in the classroom.

Possible course sequence
Example "percentages"

The whole group works with the learning material "math-starter". This leads to questions and discussions about debts, interest on debt, debt overload, …

MS **Maths in everyday life**

One group detects insecurities in the field of percentages. For detailed specification and clarification the participants work with the learning material.

Private credits, education loans
Consumer credits, loan contracts
Business debts
Taxes
Account overdraft
Mortgages
Alimonies
Health costs
Rent and mortgage rates
Leasing

What is money owed for?
county of Zurich, 2007

Extract from the learning material toolkit
part 1 "math-starter"

How old are the debtors?

Over 60 years 5%

up to 20 years 2%

31 - 40 years 31%

41 - 50 years 26%

21 - 30 years 23%

51 - 60 years 13%

Who has debts?

Single parents 8%

Single persons 42%

Couples with children 21%

Singles in flat-sharing communities 17%

Couples without children 12%

D   **Determination of Knowledge Level (Problems)**

2.   Percentages as Fractions

Percent means "in a hundred" (per-cent → per hundred).

e.g. "1% of the cars" means on car in a hundred, "2%" means two in a hundred,...

2.1   Each percentage can be written as decimal fraction.

Translate the percentage into a decimal fraction.

12% =   0,12

50% = _____

2% = _____

0,5% = _____

**Excerpt from the learning material toolkit
part 2 "Determination of Knowledge Level", Problems**

Determination in the form of a conducted discussion

Discuss on Percentages as Numbers

Discuss the following points in groups of two or more.

Where do you read percentage rates in everyday life?

Where do you use percentage rates?

A   Why can 20% stand for different numbers?

Make an example where 20% correspond to CHF 100,-

Make an example where 20% correspond to CHF 2,-

Make an example where 20% correspond to CHF 10'000.-

Why is this possible?

**Excerpt from the learning material toolkit
part 2 "Determination of Knowledge Level", conducted discussion**

I   **Improvement of Understanding**

After the insecurities have been laid open, the course managers support the course participants. Suggestions are made to improve the understanding in the form of structured problems, visualisations and models.

From percentages to fractions

Colour the percentage in the one-hundred-square and also indicate the portion as a fraction.

Example: 10 Points correspond to 10% and are 1/10 of all points

25% = _____

50% = _____

etc.

Excerpt from the learning material toolkit part 3 "Improvement of Knowledge Level"

Determine knowledge level

At a later point the participants return to the questions in the „math-starter" or solve the problems again in the determination of knowledge level and test their understanding of percentages.

Overview over the current learning sets

MS   math-starter

Title	Content
Pets	comparison of prices
Million-profits	large numbers
Prices in comparison: Lidl, Denner, Coop	tables, understand percentages
Costs of a car	budget, money, percentages
Floor plans	footage, lengths, scale

D/I   Formation of Numbers: Understand Numbers

Set	Title	Content
Set A	The decade system	number lines, place value tables
Set B	Common fractions	Parts of a whole, number lines
Set C	Decimal numbers (decimal fractions)	place value table, number lines, linking up fractions
Set D	Percentages	number lines, linking up common fractions and decimal fractions

D/I   Mathematical Operations: Understand Calculations

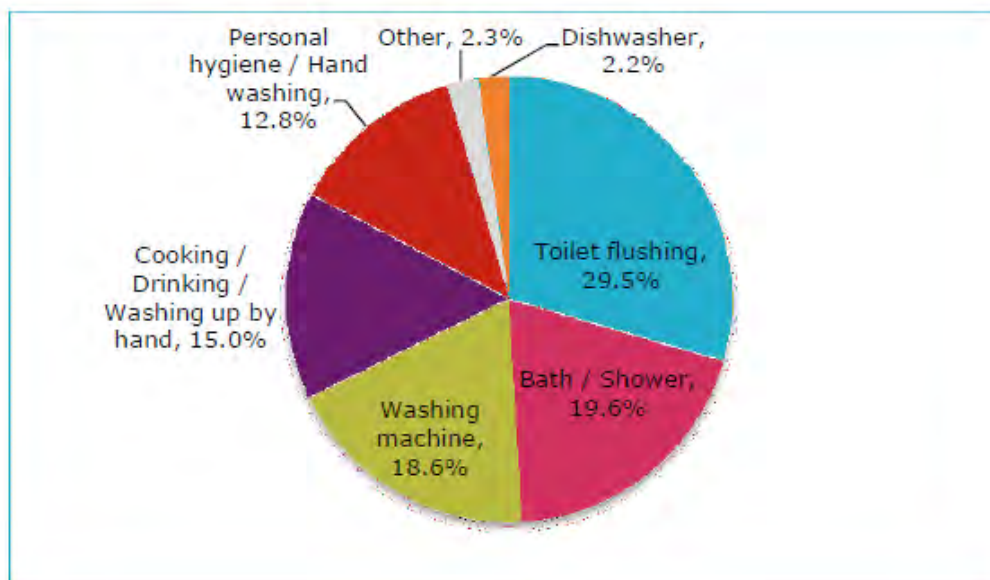| Set	Title	Content
Set E	Plus and Minus	understand operations, using the place value table, number lines
Set F	Multiplications and Divisons	understand operations, doubling and halving, inverted operations
Set G	Which Calculation Fits? Calculation Rules	recognise right operations, calculation rules: multiplications & divisions before additions & subtractions calculations in brackets (parenthesis) are done first, use commutative law doe smart calculcations

July 2014

(not broached: Minus before the bracket, use in algebraic fields)

D1 Understand Numbers – Percentages
Determination of Knowledge Level

1. What do these percentage rates mean?

- Personal hygiene / Hand washing, 12.8%
- Other, 2.3%
- Dishwasher, 2.2%
- Toilet flushing, 29.5%
- Cooking / Drinking / Washing up by hand, 15.0%
- Bath / Shower, 19.6%
- Washing machine, 18.6%

In Switzerland each person uses 201 litres of drinking water per day. Water consumption has therefore been stable within the last 30 years; since a few years it has even decreased a bit, although the number of small households has increased significantly. The reasons for the stable to slightly decreasing consumption of drinking water are faucets and bathroom facilities with smaller water-consumptions. Technology therefore actually helps save precious water. www.coopzeitung.ch (March 2012)

simple ☐ ☐ ☐ ☐ very difficult

A **For what do we use the most water?**

B **How much water do we use daily for baths/showers?**

C **Are the statements correct?**

For cooking, drinking and washing up by hand and machine we use double as much water as for flushing the toilet.

For the washing machine we need the same amount of water as for flushing the toilet.

2. Percentages as Fractions

Percent means "in a hundred" (per-cent ⧠ per hundred).

e.g. "1% of the cars" means on car in a hundred, "2%" means two in a hundred,...

simple

very difficult

2.1 **Each percentage can be written as decimal fraction.**

Translate the percentage into a decimal fraction.

12% =   0,12

50% = _____

2% = _____

0,5% = _____

_____

2.2 **50% means 50 in a hundred. That is ½ of hundred.**

Each percentage rate can be written as a common fraction. Write the

percentage rate as a fraction.

simple

very difficult

50% = 50/100 = 1/2   20% =
_____   _____

1% =   75% =
_____   _____

10% =   0,5% =
_____   _____

2.3 **Numbers can be entered as decimal fractions, (common)**

fractions or as percentages on the number line.

e.g. 0,5 = 1/2 = 50%

Enter the following percentages on the number line.

simple

very difficult

25%   75%   125%   150%   200%

	1/4	1/2	3/4	1		5/2		
0	0,25	0,5	0,75	1		2,5		3
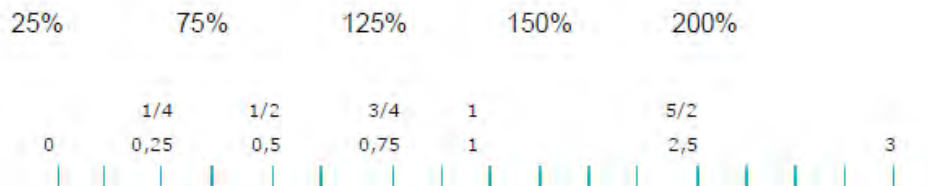
2.4 Fill the boxes.

Percentage	Decimal Fractions	Common Fraction
40%	0,4	4/10 = 2/5
80%		
		3/4
	0,01	
		1/20

3. Sales Reductions (Discounts)

3.1 Fill the boxes.

Old Price	New Price	Discount in %
CHF 100.-	CHF 90.-	
CHF 50.-	CHF 40.-	
CHF 500.-	CHF 490.-	

3.2 **On all articles a discount of 10% is granted.**

How much is the discount in CHF?

What is the new price?

10% AUF ALLES

Original Price	Discount 10%	To pay 90%
CHF 120.-		
CHF 75.-		
CHF 1500.-		

4. Percentage of ... absolute and relative

The daily energy need of an adult is indicated as 100% (relative), absolute these are 1900 kcal.

A A helping (portion) of rice covers 12% of the protein demand (Eiweiss). That is 6 g.

A helping (portion) of rice covers 12% of the protein demand (Eiweiss). That is 6 g.

B 45 g of carbohydrates (Kohlenhydrate) cover 17% of the daily need.

How many carbohydrates in gramms is that per day?

Discuss Percentage Rates in Graphs

Discuss the following points in groups of two or more.

Interpret graph, discuss

What statements are possible?

What constitutes 100%?

ARBEITSMARKT, DIE SCHWEIZ ALS AUSNAHME

Arbeitslosenrate im europäischen Durchschnitt (November 2012)

Job market, Switzerland as exception

Bar diagram

Rate of unemployment, European average (November 2012)

Entwicklung Arbeitslosenrate in der Schweiz

Line diagram

Development of unemployment rate in Switzerland

Registrierte Arbeitslose in der Schweiz seit

Total 142'300

1-6 Monaten 65,5%

7-12 Monaten 20,1%

mehr als ein Jahr 14,5%

Circle diagram

Registered unemployed persons in Switzerland since

1-6 months

7-12 months

more than a year

Quelle: Seco, eurostat

Discuss on Percentages as Numbers

Discuss the following points in groups of two or more.

Where do you read percentage rates in everyday life?
Where do you use percentage rates?

A   **Why can 20% stand for different numbers?**
Make an example where 20% correspond to CHF 100.-
Make an example where 20% correspond to CHF 2.-
Make an example where 20% correspond to CHF 10'000.-

Why is this possible?

B   **Which is the better offer?**
Offer 1 2% on all bought goods plus a reduction of 50.-
Offer 2 4% on all bought goods.

At what price is offer 1 better? When is offer 2 better?

C   **Why is the percentage rate a relative amount?**
2011 Luxemburg had
13'500 unemployed (absolute value). That is 7.4% (relative value)

2012 Switzerland had
140'000 unemployed (absolute value). That is less than 3.5% (relative value).

Compare the two countries: Which country has the higher unemployment rate?
Absolute value and relative value: what is the difference? Value Added Tax
(VAT); discounts, petrol consumption of cars, …

D   **How can percentage rates be illustrated?**

In a circle diagram always 100% are shown.

What advantages do circle diagrams have, what advantages do percentage rates in bar diagrams have?

(not broached: Increase/decrease of percentages)

Beispiel Verständnisaufbau

D2   **Understand Numbers – Percentages**
Improvement of Understanding

Percentages as Fractions

100% stands for the whole lot.
Per-cent stands for "per hundred" (lat.: centum, ital.: cento, french: cent).

The notation in percent is one of the three common notations of numbers. Like the fractions or decimal fractions (decimal numbers) the percentages indicate the portions of a whole.

Fraction	Decimal fraction (Decimal number)	Percent
1/2	0,5	50%

EDU **Sometimes calculations are easier when percentages are translated into fractions.**

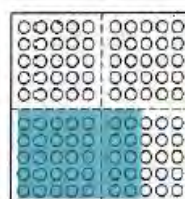E.g. "25% of the production is useless". That means a quarter of the production or every fourth piece

From percentages to fractions

Colour the percentage in the one-hundred-square and also indicate the portion as a fraction.

Example: 10 Points correspond to 10% and are 1/10 of all points

25% = _____

50% = _____

etc.
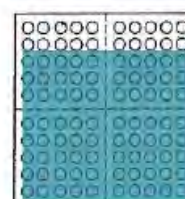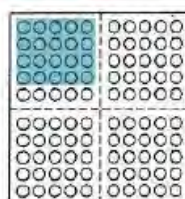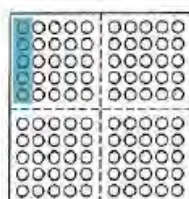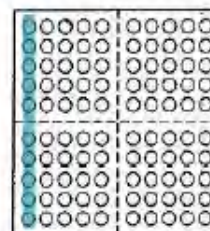
BSP Indicate the percentage and the fraction of the coloured dots.

1/5	3/10	½	3/4	1
0.2	0.3	0.5	0.75	
20%	30%	50%	75%	100%

0

Enter the positions into the number line.

1/10   20%   40%   1/4   3/4

0   1
                                                                    100%

EDU   **Illustrate percentage as fraction with the help of a circle.**

Instructions: Take to circles of the same size and make a straight cut to the middle. Slot them into each other as shown below. Automatically different segments are formed when you turn the circles. With this "percentage disc" fractions and percentages can be visualised.

(Idea from Schweizer Zahlenbuch 6)

With the percentage clock you can measure the percentage or the fraction of a segment. Copy the percentage clock onto foil so you can read the percentage or the fraction.
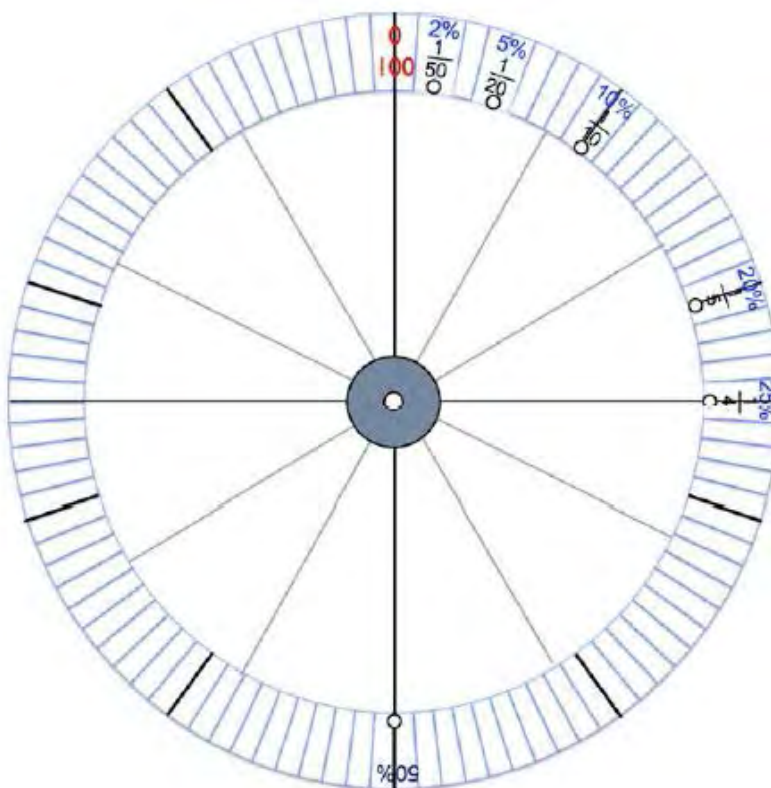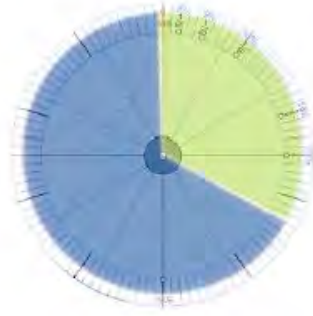
E.G. Draw a circle diagram with the portions 25%, 30%, 50%, 75% with the help of the percentage clock.

Percentages relative and absolute

Step-by-step approach to clarify the terms absolute and relative.

EDU Material: Masking tape, felt pen, piece of string, tape measure or folding rule.

1. Step: Enter percentages

Stick a piece of masking tape along the edge of a table. Take a piece of string of the same length as the table.

By folding the piece of string determine a half, a quarter, … of the length of the table. Mark the fractions and corresponding percentages on the masking tape.

z.B.   1/2 = 50%   1/4 = 25%   1/10 = 10%

   3/4 = 75%   4/5 = 80%

2. Step: Determine portion

Now measure the length of the table and initiate discussions e.g. how long 10% = 1/10, 25%= 1/4 of the table is.

Example: The table is 74 cm long.

50% of the table corresponds to a length of 37 cm

10% of the table corresponds to a length of 7.40 cm

Afterwards the result can be confirmed by measurement.

It is worthwhile to immerse yourself thoroughly into this exercise before the mechanical way of calculating percentages can be discussed.

It is helpful to use the two notations percentage and fraction at the same time. Like this the basic idea is supported "a percent is a hundredth/is the hundredth part".

Percentages are relative indications, the measurements are absolute values. (see page 9)

Calculate Reductions (Discounts)

Central question: What is the whole entity? What total number to assume?

The total is always 100%.

EDU A dress is marked at CHF 560.-
On this price a discount of 7 % is given. How much does the dress cost?

100%	▯	CHF 560
1%	▯	560 ▯ 5.6
		100
7%	▯	7 · 5.6 = 39.2
Price discount CHF		39.20
Amount due CHF		520.80

%	CHF
100	560
	5,6
7	39,2
93	520,80

A dress is marked at CHF 560.-
It is now sold for CHF 50.- less.

CHF 560	▯	100%
CHF 1	▯	100
		560 ▯ 0.18
CHF 50.-	▯	50 · 0.18 = 9
Discount	9%	

%	CHF
100	560
0,18	1
9	50

With a loyalty card a dress costs 15% less.
That is a discount of 72 CHF.
How much is the dress without a loyalty card?

%	CHF
15	72
1	4,8
100	480

15% ≙ 72 CHF

1% ≙ $\frac{72}{15}$ ≙ 4.8

100 % ≙ 100 · 4.8 = 480 CHF

85% ≙ 85 · 4.8 = 408 CHF

Price without discount CHF 480.-

Price with discount (loyalty card) CHF 408.-

Other Contexts

EDU **Easy problems should be able to be solved without a calculation formula.**

Pick examples from the personal surroundings.

E.G. 10 of the 50 participants are staying for supper.

That is
1/5 = 2/10   or
20 von 100   or
20/100 = 20%

4 out of 20 eggs are damaged.

That is
1/5 = 2/10   or
20 von 100   or
20/100 = 20%

EDU　Test critically price discounts of adverts and shops.

Check declarations to sales critically.

Look for examples where the calculations aren't correct (there are many!)

BSP　What do you say to this declaration?

Relative and Absolute Declarations in Everyday Life

Relative: Part of a whole in percent, e.g. half of the insured people, 25% of the price.

Absolute: The avtual values, e.g. CHF 560.50 cheaper.

EDU　In which cases is the relative declaration more useful, in which cases the absolute declaration?

BSP

absolute	relative
The product contains 12 g of sugar.	The product covers 14 % of the daily demand.
For the food the VAT is CHF 14.40.	On foodstuffs the VAT is 2.5%.
The price reduction is CHF 240.-.	A discount of 40% was given.

EDU **Calculate discounts**

Central question: What is the whole entity? What total number to assume?

E.G. Complete the table.

Original Price		Discount		Amount due	
absolute	relative	absolute	relative	absolute	relative
CHF 560.00	100%		7%		
				CHF 297.50	85%
CHF 20.-		CHF 3.-			
		CHF 580.-	10%		
		CHF 200.-		CHF 8'200	

Literatur

Moser Opitz, E. (2007). *Rechenschwäche – Dyskalkulie. Theoretische Klärungen und empirische Studien an betroffenen Schülerinnen und Schülern.* Bern u.a.: Haupt, Seite 222).

SECTION 3

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Burgergemeinde Bern

ALM21, the 21st Annual Conference of Adults Learning Mathematics – A Research Forum was held in Bern, Switzerland, in 2014. ALM21 was co-organised by the University of Bern, the Swiss Federal Institute for Vocational Education and Training SFIVET and the Swiss Federation for Adult Learning SVEB.

Sessions covered a wide range of issues that address and are relevant to the theme of adults learning mathematics in various contexts, including:

- Evidence based teaching
- Multilingual classrooms
- Flipped classrooms
- Vocational education and training
- Workplace based training
- Bridging different locations of learning

Furthermore, specific experiences and tools were presented, namely:

- A numeracy curriculum from a Swiss employment programme
- The French "Alternance" model
- A numeracy toolkit
- The maths everywhere app
- Maths for medical needs

Plenary presentations were made by the following:

Susan Easton, NIACE UK
Corinne Hahn, ESCP Europe
Walter Herzog, Institut für Erziehungswissenschaft, Universität Bern
Hansruedi Kaiser, Swiss Federal Institute for Vocational Education and Training SFIVET
Cäcilia Märki, Swiss Federation for Adult Learning SVEB
Mamokgethi Phakeng, University of South Africa

These proceedings include articles, PowerPoint slides for some presentations as well as abstracts of workshops or posters.